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Abstract

This paper presents a medium-scale micro-founded model with real and nominal imperfections, named "MOISE" (MOdel for the ISraeli Economy). The model was developed at the Bank of Israel to support monetary policy formulation, and builds on similar models in wide use among central banks. It includes extensions and modifications that reflect the special characteristics of the Israeli economy and are intended to improve the model's fit to Israeli data. The model was estimated using the increasingly popular Bayesian approach, using Israeli data for 1992:Q1 to 2009:Q4. The model's fit was tested using two criteria: the moments' fit and in-sample forecast performance. Finally, the paper presents and analyzes the model's properties using impulse response functions and variance decomposition.

JEL classification: C11, E32, E37, F41

Keywords: DSGE, Open Economy Macroeconomics, Bayesian Analysis, Forecasting, Business cycles, Models and Applications, Israel

מודל DSGE למשק הישראלי

דוד אלקיים, איל ארגוב, אליעזר בורנשטיין, אלון בנימיני, עמנואל ברנע ועירית רוזנשטרום

תקציר

מאמר זה מציג מודל מאקרו כלכלי, המבוסס על יסודות מיקרו כלכליים ומכיל חיכוכים ריאליים ונומינליים. המודל פותח בבנק ישראל לשם תמיכה בניהול המדיניות המוניטארית, והוא מושתת על עקרונות של מודלים דומים, המשמשים בבנקים מרכזיים בעולם. במודל נכללו הרחבות והתאמות המבטאות מאפיינים ייחודיים לישראל, במטרה לשפר את התאמתו לנתוני המשק הישראלי. המודל נאמד בשיטת אמידה ביסייאנית לתקופה 1992:Q1-2009:Q4. טיב ההתאמה של המודל נבחן על פי שני קריטריונים – התאמת המומנטים וטעויות התחזית בתוך המדגם. כמו כן אנו מנתחים את תכונות המודל באמצעות בחינה של תגובות לזעזועים (impulse responses) ובאמצעות פירוק השונות של טעויות התחזית (variance decomposition).

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1 Introduction

During the last decade there has been a growing acknowledgement of the important role of macroeconomic models in the conduct of monetary policy, alongside the adoption of inflation-targeting regimes in many countries. These models provide a theoretical framework for a central bank's economic discussions and analysis, and help economists to assess the current state of the economy and produce forecasts. Perhaps most importantly, they allow policy makers to analyze and quantify the effects of alternative economic scenarios and policy measures.

Following the influential work of Woodford (2003), Christiano et al. (2005) and Smets and Wouters (2003), New Keynesian (NK) Dynamic Stochastic General Equilibrium (DSGE) models became the main class of macroeconomic models used by central banks, as well as in academia. These models are built on microeconomic foundations that include optimizing agents and general equilibrium conditions, as in the basic Neo-Classical RBC model presented by Kydland and Prescott (1982). However, in order to allow for the non-trivial effects of monetary policy, the NK school added three ingredients to the basic RBC model: (1) money as a unit of account (2) monopolistic competition and (3) nominal rigidities. One outcome of this setup is that in the short to medium runs, nominal shocks affect real variables. Another important property of these models is that expectations affect the current behavior of households and firms and therefore the management of expectations becomes an important channel through which monetary policy operates.

Two leading examples of small-open-economy DSGE models that have been adopted by many central banks in the western world are the ECB's NAWM[1] and the Riksbank's RAMSES[2]. The widespread adoption of these models was the result not only of progress in economic theory, but also advances in econometric practice. Specifically, the reintroduction of Bayesian methods into macroeconomics, made possible by increased computer power,

[1] New Area Wide Model (see Christoffel et al. (2008)).

[2] Riksbank Aggregate Macromodel for Studies of the Economy of Sweden (see Adolfson et al. (2007)).

enabled the estimation of models that previously could only be calibrated.

This paper presents a small open economy DSGE model for the analysis of the Israeli economy (MOISE), which is employed by the Bank of Israel's Research Department. The specification of MOISE follows that of RAMSES and especially NAWM, both of which specify the structure of the economy in terms of both real and nominal variables. On the real side, these models determine labor input and the main components of the national accounts data, i.e. GDP, consumption, investment, exports and imports. On the nominal side, the models specify the dynamics of prices (the CPI and various deflators), wages, the exchange rate and the interest rate set by the central bank. They contain real rigidities, such as habits in consumption and adjustment costs in investment and exports. They also include nominal rigidities such as Calvo (1983) style price and wage stickiness and indexation. These rigidities contribute significantly to the model's fit and, as mentioned above, the nominal rigidities establish a link between the nominal and real sides of the economy.

In order to adjust the model to the characteristics of the Israeli economy, we introduce some modifications into MOISE that depart from NAWM. For example, Israel is a much smaller and more open economy than the Euro area; about 70 percent of Israel's imports of goods are raw materials used in the production of final goods. Therefore, we added an import component to the production of exports and government consumption goods and assumed local currency pricing in both exports and imports. To cope with the non-stationarity of the real interest rate during the sample period, we added a highly inertial risk-premium process that affects the long-run real interest rate. We also introduced (exogenously determined) investment in inventories, with the intention of bridging the gap between the theoretical model's resource constraint and the corresponding national accounts identity, while using fixed capital formation data as the observed investment component.

The structural parameters of MOISE are estimated using quarterly Israeli data on 24 variables for 1992 to 2009. The estimation uses full-information Bayesian techniques as in

An and Schorfheide (2007). A common problem in estimating DSGE models is that the model typically assumes balanced growth, which is not evident in the data. We cope with this difficulty by extending the observation block to include an "additive hybrid model" for a model-consistent extraction of idiosyncratic trends in the real variables. The shocks of the additive hybrid model absorb trends that cannot plausibly be explained by the balanced growth model. The advantage of this setup, as opposed to pre-filtering the data, is that it allows the Kalman filter to use the information contained in all the data series simultaneously in order to divide the variation in the data between dynamics that are well-explained by the theoretical model and those that are not. The additive hybrid model framework is also used to identify and filter out yield-curve term premiums.

We find the estimation results of MOISE to be satisfactory in the sense that the likelihood has curvature with respect to most parameters and the posterior mode/mean lie within reasonable territories, even for parameters with relatively wide priors. The model's fit to the data is evaluated by means of two tests: a comparison of the model's implied second moments, namely variances and correlations, to the corresponding moments in the data and a comparison of the model's in-sample forecast errors to those of naive and BVAR models. The two tests show the model to be consistent with the data. Thus, most model-based moments are not significantly different from the observed ones; the model's (unconditional) forecasting performance is not worse than that of naive and BVAR models; and the model even out-performs other models in forecasting the interest rate.

The paper is organized as follows: section 2 briefly describes the role of DSGE models in central bank policy formulation. Section 3 presents the structure of the model. In section 4 we prepare the model for estimation and present the estimation results and in section 5 we evaluate the model's fit to the data. In section 6, we present the contribution of the structural shocks to the observed fluctuations in macroeconomic variables (through forecast-error-variance decompositions) and analyze the dynamics following various structural shocks (using impulse response functions). Finally, section 7 offers some concluding remarks.

2 The role of DSGE models in central bank policy formulation

Theoretically-based models are designed to achieve a wide range of objectives. In this section, we discuss the three main roles of DSGE models in central bank policy making, which guided the specification of the current model.

a. To provide a basis for discussion

A macroeconomic model describes the relationships among the various sectors of the economy and the main factors that are believed to drive the economy. In other words, a macroeconomic model embodies our knowledge and understanding of the economy. Any macroeconomic model, no matter how large or complex, is only a rough simplification of reality, and model builders and users are well aware of this fact. Nevertheless, a macroeconomic model summarizes the way in which we view the economy and, as such, can serve as a common platform for economic discussion. Even if not necessarily agreed upon by all sides, it facilitates effective communication, both within a central bank and between it and the public.

b. Nowcasting and forecasting

Another important use of DSGE models is in assessing the current state of the economy and producing forecasts. The data for many variables, especially labor market and national accounts data, is published with a lag of several months. A macroeconomic model makes it possible to "forecast" the current level of such variables (that is, to "nowcast" them) and to assess the phase of the business cycle. These assessments, combined with assumptions regarding the future paths of various exogenous variables, make it possible to forecast the model's endogenous variables. This process (of nowcasting and forecasting) incorporates information external to the model, as well as judgment.

c. To evaluate alternative policy measures and economic scenarios

The specification of theoretically-based and empirically-validated models facilitates economic analysis and helps us to understand the forces that drive the economy, while taking into consideration the simultaneous relationships between the various sectors of the economy. In particular, the model enables us to analyze the effect of alternative policy measures and different assumptions, e.g. with respect to exogenous variables. In an inflation-targeting regime the central bank decides on its policy, mainly by setting the level of the interest rate, in order to achieve its inflation target (alongside other objectives). Being micro-founded, the model enables the central bank to assess the effect of its alternative policy choices on the future paths of the economy's endogenous variables, in a way that is immune to the Lucas (1976) critique. Without such a model, it would be difficult to forecast even the direction of a policy effect on some endogenous variables. However, a well-specified and estimated macroeconomic model makes it possible to quantify the effect of different policy measures and of exogenous variables/shocks. The model also allows us to calculate confidence intervals for the forecast variables as well as to assess the risks associated with the economic outlook.

3 The model

The model follows the lines of the ECB's NAWM and the Riksbank's RAMSES (see Christoffel et al. (2008) and Adolfson et al. (2007), respectively).[3] The economic agents in the model include households, firms of several types in the production sector, government and an inflation-targeting central bank whose policy tool is the nominal interest rate. The production sector includes monopolistic producers of intermediate goods (who employ labor and capital as production inputs), competitive producers of final goods, importers and exporters.

[3] Other similar central bank models include Brubakk et al. (2006) for Norway, Murchison and Rennison (2006) for Canada, Beneš et al. (2009) for New Zealand and Seneca (2010) for Iceland.

This section presents the main equations describing the structure of the economy, as well as the derived optimality and equilibrium conditions. Since most of the model is not innovative, explanations are brief except when the model deviates from common practice.

3.1 Households

3.1.1 Preferences and the budget constraint

The model consists of a continuum of households, indexed by $h \in [0,1]$. Households derive their lifetime utility from the discounted flow of private consumption (with external habit formation) and leisure:

$$E_t \sum_{k=0}^{\infty} \left[\beta^k \left(\varepsilon^C_{t+k} \ln \left(C_{h,t+k} - \kappa C_{t+k-1} \right) - \frac{\varepsilon^N_{t+k}}{1+\zeta} \left(N_{h,t+k} \right)^{1+\zeta} \right) \right], \tag{1}$$

where E_t is the mathematical expectations operator, $C_{h,t}$ denotes the consumption composite consumed by household h in period t and $N_{h,t}$ denotes working hours. The parameter β is the discount factor and ζ is the inverse of the Frisch elasticity of labor supply. Households are subject to external habit persistence, where the parameter κ measures its degree and C_t denotes the composite of *aggregate* consumption in period t. Households' preferences are subject to two exogenous shocks: ε^C_t which is a consumption-demand shock and ε^N_t which is a labor supply shock.[4]

The period-by-period budget constraint faced by household h is given by:

$$\begin{aligned}
& \left(1 + w_{\tau^C} \tau^C_t\right) P_{C,t} C_{h,t} + P_{I,t} I_{h,t} + P_{I,t} \Delta INV_t \\
& + \left(\varepsilon^{RP}_t \varepsilon^{DRP}_t R_t\right)^{-1} B_{h,t+1} + \left(\varepsilon^{RP}_t \Gamma_{B^*,t} R^*_t\right)^{-1} S_t B^*_{h,t+1} + \Xi_t + \Upsilon_{h,t} \\
= \; & \left(1 - \tau^N_t - \tau^{W_h}_t\right) W_{h,t} N_{h,t} + \left(1 - \tau^K_t\right) \left[R_{K,t} u_{h,t} - \Gamma_u \left(u_{h,t}\right) P_{I,t}\right] K_{h,t} \\
& + \tau^K_t \delta P_{I,t} K_{h,t} + \left(1 - \tau^D_t\right) D_{h,t} - T_t + B_{h,t} + S_t B^*_{h,t} \; .
\end{aligned} \tag{2}$$

The first term, $\left(1 + w_{\tau^C} \tau^C_t\right) P_{C,t} C_{h,t}$, denotes nominal expenditure on consumption, τ^C_t is the rate of value added tax (VAT), w_{τ^C} is the share of goods subject to VAT and

[4] In general, we will assume that shocks follow a log-AR(1) process. For example, $\log(\varepsilon^C_t) = \rho^C \log(\varepsilon^C_{t-1}) + \eta^C_t$, where η^C_t is a white noise process.

$P_{C,t}$ is the pre-tax price of the consumption good. The term $P_{I,t}I_{h,t}$ is the expenditure on fixed capital investment and $P_{I,t}\Delta INV_t$ is the expenditure associated with the change in inventories. The latter is exogenously determined (and identical across all households, hence the subscript h is omitted) and is empirically motivated (it is needed to satisfy the aggregate resource constraint in a way that will be consistent with national accounts data).[5] We assume that the change in inventories, as a share of GDP ($\Delta inv_t = \frac{\Delta INV_t}{Y_t}$), follows an AR(1) exogenous process:

$$\Delta inv_t = \rho_{\Delta INV}\Delta inv_{t-1} + (1-\rho_{\Delta INV})\,\Delta inv + \eta_t^{\Delta INV}. \tag{3}$$

In the second row of the budget constraint (2), $B_{h,t}$ and $B^*_{h,t}$ denote bond holdings at the beginning of period t, denominated in domestic and foreign currencies, respectively. The market price of the local currency bond, $\left(\varepsilon_t^{RP}\varepsilon_t^{DRP}R_t\right)^{-1}$, is driven by the short-term gross nominal interest rate set by the central bank, R_t, and by two premium shocks that drive a wedge between market return on bonds and the risk-free central bank rate. The first shock, ε_t^{RP}, which also drives the price of foreign currency bonds,[6] is introduced so as to generate a correlated shift in demand for both consumption and investment. The second shock, ε_t^{DRP}, drives the price of the domestic-currency bond only. It is introduced in order to account for the time-varying (and even non-stationary) long-term real yields on inflation-indexed domestic treasury bonds during the sample period (see section 4.1). For

[5] A common practice in addressing this issue is to add a measurement error linking observed GDP to its model counterpart (see Christoffel et al. (2008), among others). This approach, however, ignores the fact that a change in inventories requires additional resources—both imported and domestically-produced inputs. This, in turn, has implications for monetary policy.

[6] Hence, it is labeled as a 'symmetric' shock and can be thought of as a reduced form of some type of financial intermediation premium.

this purpose, we essentially assume a unit-root process:[7]

$$\varepsilon_t^{DRP} = 0.99\varepsilon_{t-1}^{DRP} + (1-0.99) + \eta_t^{DRP}.$$

With respect to the foreign-currency bonds, S_t denotes the nominal exchange rate, while R_t^* is the foreign risk-free nominal interest rates. We assume an external financial intermediation premium associated with these bonds, given by:

$$\Gamma_{B^*} = \varepsilon_t^{RP^*} \exp\left[-\gamma_{B^*} s_{B^*,t+1} - \gamma_S E_t \left(\frac{S_{t+1}}{S_t}\frac{\bar{\Pi}_{t+1}^*}{\bar{\Pi}_{t+1}}\frac{S_t}{S_{t-1}}\frac{\bar{\Pi}_t^*}{\bar{\Pi}_t} - 1\right)\right], \tag{4}$$

where $s_{B^*,t+1} \equiv \left(S_t B_{t+1}^*\right) / \left(P_{Y,t} Y_t\right)$ is the ratio of total net foreign assets to nominal GDP. Assuming an endogenous premium that depends on $s_{B^*,t+1}$, as specified in (4), ensures a stable (non-stochastic) steady state (with a zero net foreign asset position).[8] Following Adolfson et al. (2008), we assume that the premium also depends on the expected nominal depreciation, in order to allow for some sluggishness in the dynamics of the real exchange rate. However, in order to account for the disinflation process in Israel during the 1990s, we assume that this risk premium is not driven by depreciation *per se*, as in Adolfson et al. (2008), but by depreciation adjusted for the inflation target differential, $\bar{\Pi}_t^*/\bar{\Pi}_t$. Finally, $\varepsilon_t^{RP^*}$ is an exogenous shock to the external premium.

To complete the expenditure side, Ξ_t denotes lump-sum transfers and $\Upsilon_{h,t}$ denotes household h's holding of state-contingent securities that provide insurance against household-specific labor income risk. The latter provides analytical convenience since $\Upsilon_{h,t}$ guarantees that, despite the heterogeneity in wages and labor services across households, all households choose identical allocations in equilibrium (as in Christiano et al. (2005), among others).

Households provide labor services at an hourly wage rate of $W_{h,t}$. A household's labor income is subject to two taxes: a direct income tax τ_t^N and a social security tax $\tau_t^{W_h}$.

[7]Such a highly inertial shock drives the real forward rates for longer terms (hence this shock may be thought of as a shock to the "natural" interest rate). In turn, we will later assume that these forward rates serve as an anchor in the central bank's policy rule (see the interest-rate rule specified by equation 62 below). Thus, the nominal interest rate eventually adjusts so as to offset the effect of the shock on the market rate, and therefore the effect of the shock on consumption and investment persists only in the short to medium run.

[8]See Schmitt-Grohe and Uribe (2003).

The household also has capital income, where $R_{K,t}$ denotes the nominal price of capital services, $u_{h,t}$ denotes the intensity of capital utilization and $K_{h,t}$ denotes the capital stock owned by household h. The tax rate on net capital income is τ_t^K and there are two costs associated with capital services: a cost associated with utilization intensity, $\Gamma_u\left(u_{h,t}\right)$ (which is specified below), and depreciation at a rate of δ.

Household h earns a flow of dividends, $D_{h,t}$, deriving from its ownership of monopolistic firms. τ_t^D is the tax rate on dividend income. Finally, the variable T_t denotes a lump sum tax.

3.1.2 The consumption and saving decision

We define $\beta^k \cdot \Lambda_{h,t+k}/P_{C,t+k}$ (for $k \geq 0$) as the Lagrange multiplier on the budget constraint (2). The first-order conditions with respect to $C_{h,t}$, $B_{h,t+1}$ and $B^*_{h,t+1}$ obtained from the maximization of the utility function (1) subject to the budget constraint (2) are as follows:

$$\Lambda_{h,t} = \varepsilon_t^C \frac{\left(C_{h,t} - \kappa C_{t-1}\right)^{-1}}{1 + w_{\tau^C}\tau_t^C}, \tag{5}$$

$$\beta\varepsilon_t^{DRP}\varepsilon_t^{RP} R_t E_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\frac{P_{C,t}}{P_{C,t+1}}\right] = 1 \tag{6}$$

and

$$\beta\varepsilon_t^{RP}\Gamma_{B^*,t} R_t^* E_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\frac{P_{C,t}}{P_{C,t+1}}\frac{S_{t+1}}{S_t}\right] = 1. \tag{7}$$

3.1.3 Investment and capital utilization

We assume the following specification for the cost of capital utilization:

$$\Gamma_u\left(u_{h,t}\right) = \gamma_{u,1}\left(u_{h,t} - 1\right) + \frac{\gamma_{u,2}}{2}\left(u_{h,t} - 1\right)^2, \tag{8}$$

where $\gamma_{u,1}, \gamma_{u,2} > 0$.

The stock of physical capital evolves as follows:

$$K_{h,t+1} = (1-\delta) K_{h,t} + \varepsilon_t^I \left[1 - \Gamma_I\left(\widetilde{I}_{h,t}\right)\right] I_{h,t}, \tag{9}$$

where ε_t^I is an investment-specific technology shock *a′ la* Greenwood et al. (1997). The variable $\Gamma_I\left(\widetilde{I}_{h,t}\right)$ is an investment adjustment cost, associated with deviations of investment growth rates from the long-run productivity growth rate, g_z:

$$\Gamma_I\left(\widetilde{I}_{h,t}\right) = \frac{\gamma_I}{2}\left(\widetilde{I}_{h,t} - g_z^{\left(1+\omega_{\Gamma_I}\right)}\right)^2, \tag{10}$$

where

$$\widetilde{I}_{h,t} \equiv \frac{I_{h,t}}{I_{h,t-1}}\left(\frac{I_{h,t-1}}{I_{h,t-2}}\right)^{\omega_{\Gamma_I}}.$$

Note that we allow for *two* lags of investment in the cost function rather than the standard one.[9] This allows for negative serial correlation in the growth rate of investment that appears to be present in the data.

We define $\beta^k \Lambda_{h,t+k} Q_{h,t+k}$ (for $k \geq 0$) as the Lagrange multiplier on the capital accumulation process (9). The variable $Q_{h,t}$ has an intuitively appealing interpretation as the price of installed capital in terms of the consumption good, i.e. Tobin's Q. The resulting first-order conditions with respect to $I_{h,t}$, $K_{h,t+1}$ and $u_{h,t}$ are:

$$\begin{aligned}\frac{P_{I,t}}{P_{C,t}} &= Q_{h,t}\varepsilon_t^I\left[1 - \Gamma_I\left(\widetilde{I}_{h,t}\right) - \Gamma_I'\left(\widetilde{I}_{h,t}\right)\widetilde{I}_{h,t}\right] \\ &\quad + \left(1-\omega_{\Gamma_I}\right)\beta E_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} Q_{h,t+1}\varepsilon_{t+1}^I \Gamma_I'\left(\widetilde{I}_{h,t+1}\right)\widetilde{I}_{h,t+1}\frac{I_{h,t+1}}{I_{h,t}}\right] \\ &\quad + \omega_{\Gamma_I}\beta^2 E_t\left[\frac{\Lambda_{h,t+2}}{\Lambda_{h,t+1}}\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} Q_{h,t+2}\varepsilon_{t+2}^I \Gamma_I'\left(\widetilde{I}_{h,t+2}\right)\widetilde{I}_{h,t+2}\frac{I_{h,t+2}}{I_{h,t}}\right],\end{aligned} \tag{11}$$

$$Q_{h,t} = \beta E_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\left(\begin{array}{c}\left(1-\tau_{t+1}^K\right)\left[\frac{R_{K,t+1}}{P_{C,t+1}} u_{h,t+1} - \Gamma_u\left(u_{h,t+1}\right)\frac{P_{I,t+1}}{P_{C,t+1}}\right] \\ + \tau_{t+1}^K \delta \frac{P_{I,t+1}}{P_{C,t+1}} + (1-\delta) Q_{h,t+1}\end{array}\right)\right], \tag{12}$$

[9] The standard specification is nested in ours as the special case where $\omega_{\Gamma_I} = 0$.

and

$$R_{K,t+1} = \Gamma'_u \left(u_{h,t}\right) P_{I,t} \ . \tag{13}$$

3.1.4 Labor supply and wage setting

Households are monopolistic suppliers of differentiated labor services, $N_{h,t}$, and nominal hourly wages are staggered. Thus, following the Calvo (1983) setup, $(1-\xi_W) \in (0,1)$ is the probability of receiving an exogenous and idiosyncratic signal which leads to wage reoptimization. When there is no signal, which occurs with probability ξ_W, the h'th household updates its hourly wage according to the following indexation scheme:

$$W_{h,t} = \left(g_{z,t-1}\right)^{\chi_{W,g_z}} \left(g_z\right)^{\left(1-\chi_{W,g_z}\right)} \Pi^{\dagger}_{C,t} W_{h,t-1}, \tag{14}$$

where $\Pi^{\dagger}_{C,t} \equiv \left(\Pi_{C,t-1}\right)^{\chi_W} \left(\bar{\Pi}_t\right)^{(1-\chi_W)}$, $\Pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$ and $\bar{\Pi}_t$ is the (gross) inflation target. The variable $g_{z,t-1}$ denotes the (gross) growth rate of labor productivity and the parameter g_z is the long-run rate. The degree of indexation to productivity and to inflation in wage setting is represented by the parameters χ_{W,g_z} and χ_W, respectively.

Upon receiving an idiosyncratic signal, household h reoptimizes its hourly wage, $W_{h,t}$, in order to maximize the utility function (1) subject to the budget constraint (2) and the labor demand function discussed below (equation 21). In reoptimizing their wages, households also take into account the Calvo (1983)-style rigidity and the indexation scheme (14). All reoptimizing households in period t will have the same new optimal wage, denoted by $\tilde{W}_t$, satisfying the following first-order condition:[10]

$$E_t \sum_{k=0}^{\infty} \left\{ \left(\xi_W \beta\right)^k \left[\Lambda_{t+k} \left(1 - \tau^N_{t+k} - \tau^{W_h}_{t+k}\right) \left(g_{z;t-1,t+k-1}\right)^{\chi_{W,g_z}} \left[\left(g_z\right)^{1-\chi_{W,g_z}}\right]^k \frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}} \frac{\tilde{W}_t}{P_{C,t}} \right.\right.$$
$$\left.\left. - \varphi^W_{t+k} \varepsilon^N_{t+k} \left(N_{h,t+k}\right)^{\zeta} \right] N_{h,t+k} \right\} = 0, \tag{15}$$

[10] The holding of state-contingent securities, $\Upsilon_{h,t}$, ensures that all reoptimizing households in period t choose the same new wage.

where for every $k \geq 1$, the terms $g_{z;t,t+k} \equiv \prod_{s=1}^{k} g_{z,t+s}$ and $\Pi^{\dagger}_{C;t,t+k} \equiv \prod_{s=1}^{k} \Pi^{\chi_W}_{C,t+s-1} \bar{\Pi}^{1-\chi_W}_{t+s}$ (where $\Pi_{C;t,t+k} \equiv \prod_{s=1}^{k} \Pi_{C,t+s}$) represent the adjustment of wages to the accumulated growth of labor productivity and accumulated inflation respectively, as defined by the indexation scheme (14). For $k = 0$, we simply substitute $g_{z;t,t} = \Pi^{\dagger}_{C;t,t} = \Pi_{C;t,t} \equiv 1$.

Using the expression for the aggregate wage composite (equation 22 which is discussed below) and the law of large numbers, we are able to derive the dynamics of the aggregate wage composite:

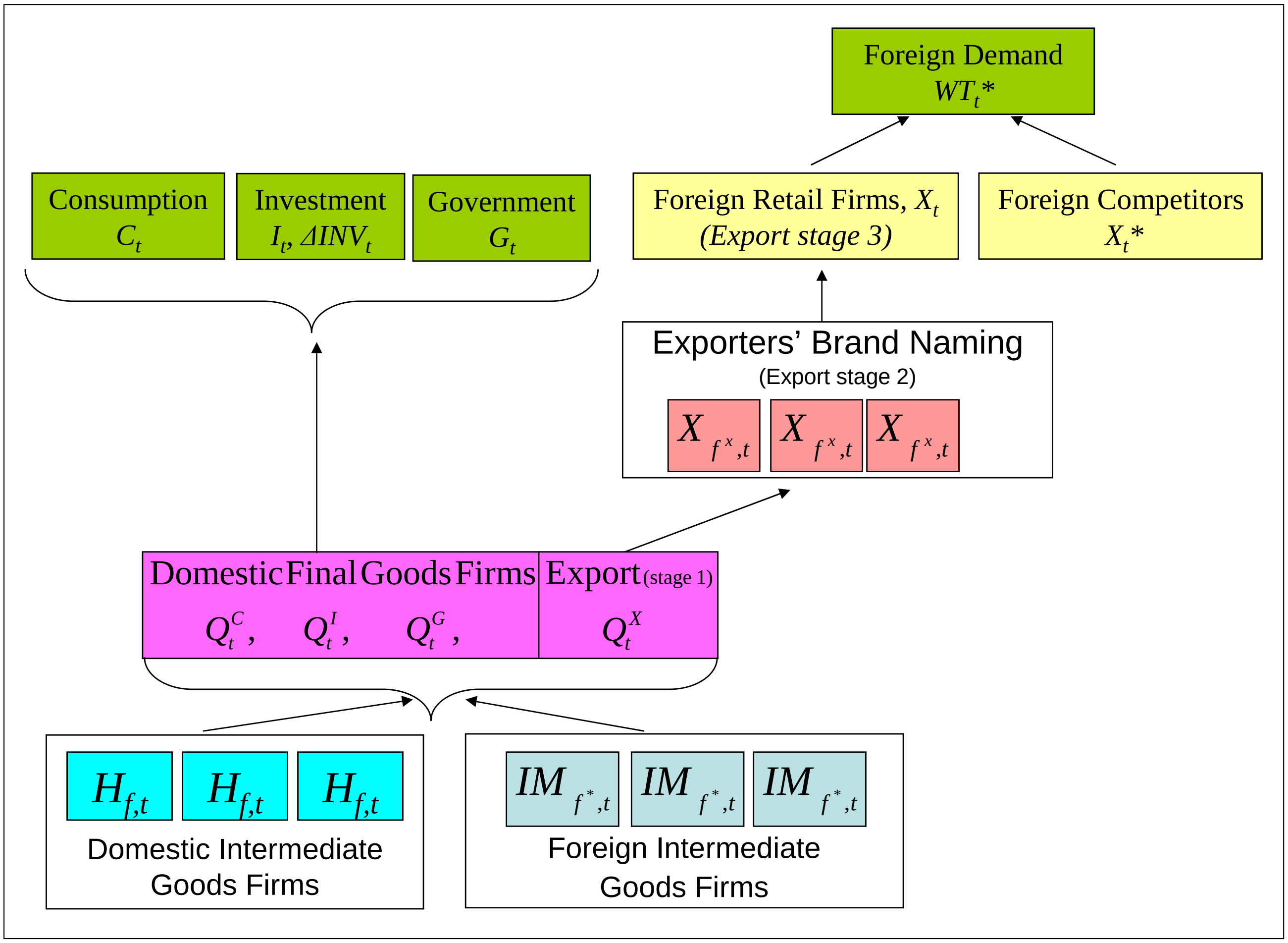
$$W_t = \left(\xi_W \left[(g_{z,t-1})^{\chi_{W,g_z}} (g_z)^{1-\chi_{W,g_z}} \Pi^{\dagger}_{C,t} W_{h,t-1} \right]^{\frac{1}{1-\varphi^W_t}} + (1 - \xi_W) \left(\tilde{W}_t \right)^{\frac{1}{1-\varphi^W_t}} \right)^{1-\varphi^W_t} . \quad (16)$$

3.2 Firms

Figure 1 illustrates the structure of the production sector, which is comprised of five types of firms:

- Monopolistically competitive domestic firms which produce differentiated intermediate goods, $H_{f,t}$, where $f \in [0, 1]$.
- Monopolistically competitive foreign firms which produce differentiated intermediate goods, $IM_{f^*,t}$, where $f^* \in [0, 1]$. These goods are imported to the domestic economy.
- Perfectly competitive firms which produce final goods for consumption, investment, government consumption and export (Q^C_t, Q^I_t, Q^G_t, and Q^X_t, respectively). The production inputs of these firms are the differentiated intermediate goods, both domestically produced ($H_{f,t}$) and imported ($IM_{f^*,t}$).
- Monopolistically competitive exporters who buy the final homogenous domestic export good (Q^X_t) and differentiate (i.e. brand name) it. The differentiated good, $X_{f^X,t}$ where $f^x \in [0, 1]$, is then sold to foreign retail firms.
- Foreign retail firms which combine the differentiated export goods ($X_{f^X,t}$) into a homogenous exported good (X_t).

Figure 1: The Structure of the Production Sector



We assume that all monopolistically competitive firms are subject to Calvo-style (Calvo (1983)) price rigidity in terms of the *local* (i.e. consumer) currency. The structure of the exporting sector is designed so as to introduce imported inputs in the production of exports and at the same time to allow for consumer-currency price rigidity in exports. We will now turn to a detailed description of each firm type.

3.2.1 Domestic intermediate goods firms

A continuum of domestic firms, indexed by $f \in [0, 1]$, produce differentiated intermediate goods, $H^s_{f,t}$. The production technology combines capital, $K^s_{f,t}$, and differentiated labor services hired from households, $N_{f,t}$:

$$H^s_{f,t} = \max\left[\varepsilon_t \left(K^s_{f,t}\right)^{\alpha} \left(z_t N_{f,t}\right)^{1-\alpha} - \psi z_t,\ 0\right]. \tag{17}$$

ε_t is a transitory technology shock and z_t is a difference-stationary labor-augmenting productivity shock that determines the balanced growth path of all real variables (both of which are symmetric across firms). The gross growth rate of the labor productivity shock, $g_{z,t} \equiv z_t/z_{t-1}$, follows an AR(1) process:

$$g_{z,t} = \left(1-\rho_{g_z}\right) g_z + \rho_{g_z} \cdot g_{z,t-1} + \eta^{g_z}_t. \tag{18}$$

The variable $K^s_{f,t}$ is (homogenous) capital services rented under perfect competition. Labor services employed by the $f'th$ firm, $N_{f,t}$, is given by a Dixit and Stiglitz (1977) Constant Elasticity of Substitution (CES) composite of household-specific labor inputs, $N^h_{f,t}$:

$$N_{f,t} = \left(\int_0^1 \left(N^h_{f,t}\right)^{\frac{1}{\varphi^W_t}} dh\right)^{\varphi^W_t}. \tag{19}$$

The exogenous CES between differentiated labor services is defined to be $\varphi^W_t / \left(\varphi^W_t - 1\right) > 1$, where $\varphi^W_t > 1$ may be interpreted as an exogenous wage markup shock. We assume the following AR(1) process for the markup shock:[11]

$$\log(\varphi^W_t) = (1-\rho_W)\log(\varphi^W) + \rho_W \cdot \log(\varphi^W_{t-1}) + \eta^W_t. \tag{20}$$

Finally, the production technology (17) includes a fixed cost term ψz_t, where the parameter ψ is calibrated to ensure zero profits in the steady state. This is consistent with the assumption of no entry or exit of firms in the steady state.

Resource allocation

Cost minimization leads to the following equation for $N^h_{f,t}$, the demand by the f'th firm

[11] All markups in the model follow AR(1) processes with this structure.

for the labor services of the h'th household:

$$N_{f,t}^{h}=\left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}}N_{f,t}. \tag{21}$$

The aggregate wage index, $W_t \equiv \frac{1}{N_{f,t}}\int_0^1 W_{h,t}\cdot N_{f,t}^{h}\cdot dh$, is given by:

$$W_t=\left(\int_0^1 \left(W_{h,t}\right)^{\frac{1}{1-\varphi_t^W}}dh\right)^{1-\varphi_t^W}. \tag{22}$$

Total variable production cost is given by :

$$TVC_t=R_{K,t}K_{f,t}^{S}+R_t^F\left(1+\tau_t^{W_f}\right)W_tN_{f,t}, \tag{23}$$

where $\tau_t^{W_f}$ is the rate of the social security tax levied on firms. Following Adolfson et al. (2007), we allow for a working capital channel, $R_t^F=1+\nu^F\left(R_t-1\right)$, where each firm borrows a fraction ν^F of its wage bill ahead of production at an interest rate of R_t.

Cost minimization leads to the following optimal allocation of resources:

$$\frac{R_{K,t}}{R_t^F\left(1+\tau_t^{W_f}\right)W_t}=\frac{\alpha}{(1-\alpha)}\frac{N_{f,t}}{K_{f,t}^{S}}. \tag{24}$$

Nominal marginal cost (MC_t) is identical across firms, as it depends only on the market prices of inputs and not on the quantities employed by the individual firm:

$$MC_t=\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}\frac{1}{\varepsilon_t z_t^{1-\alpha}}\left(R_{K,t}\right)^{\alpha}\left[R_t^F\left(1+\tau_t^{W_f}\right)W_t\right]^{1-\alpha}. \tag{25}$$

Price setting

We assume sluggish price adjustment in the domestic intermediate goods sector, based on the setup suggested by Calvo (1983). Thus, as in the staggered wages framework presented in section 3.1.4, the probability that a firm does not receive an exogenous and idiosyncratic reoptimization signal is ξ_H, in which case the firm adjusts its price according to the following indexation scheme:

$$P_{H,f,t} = \Pi_{H,t-1}^{\chi_H} \overline{\Pi}_t^{1-\chi_H} P_{H,f,t-1} \ , \tag{26}$$

where $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ and $\overline{\Pi}_t$ is the gross time-varying inflation objective. The parameter χ_H determines the degree of indexation to past aggregate domestic inflation. Upon receiving an idiosyncratic reoptimization signal, the firm adjusts its price so as to maximize the discounted sum of expected gross profits:

$$\max E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \xi_H^k \left(P_{H,f,t+k} H_{f,t+k} - MC_{t+k} H_{f,t+k} \right) \right]. \tag{27}$$

The discount factor, $\Lambda_{t,t+k} \equiv \beta^k E_t \left[\frac{\Lambda_{h,t+k}}{\Lambda_{h,t}} \frac{P_{C,t}}{P_{C,t+k}} \right]$, reflects the discounted contribution to utility, and is multiplied by ξ_H^k, the probability that the price is not reoptimized k periods ahead. The total variable cost in period t may be expressed as $MC_t H_{f,t}$ since marginal cost is invariant to the firm's own output. Taking into account the price indexation scheme (26) and the demand for differentiated intermediate goods (given by equation (39) which is discussed below) all reoptimizing firms choose the same new price, $\widetilde{P}_{H,t}$, according to the following optimality condition:

$$E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \xi_H^k \left(\Pi_{H,t,t+k}^{\dagger} \widetilde{P}_{H,t} - \varphi_{t+k}^{H} MC_{t+k} \right) H_{f,t+k} \right] = 0, \tag{28}$$

where $\Pi_{H,t,t+k}^{\dagger} \equiv \prod_{s=1}^{k} \left(\Pi_{H,t+s-1}^{\chi_H} \overline{\Pi}_{t+s}^{1-\chi_H} \right)$ for $k \geq 1$ and $\Pi_{H,t,t+k}^{\dagger} \equiv 1$ for $k = 0$.

Using the result for the aggregate price index (equation (41) below) and the law of large numbers, we derive the following price dynamics:

$$P_{H,t} = \left[(1-\xi_H) \left(\widetilde{P}_{H,t} \right)^{\frac{1}{1-\varphi_t^H}} + \xi_H \left(\Pi_{H,t-1}^{\chi_H} \overline{\Pi}_{H,t}^{1-\chi_H} P_{H,t-1} \right)^{\frac{1}{1-\varphi_t^H}} \right]^{1-\varphi_t^H}. \tag{29}$$

3.2.2 Foreign intermediate goods firms

A continuum of foreign firms, indexed by $f^* \in [0,1]$, produce differentiated intermediate goods, $IM_{f^*,t}$, which are imported to the domestic economy. We assume consumer-currency pricing subject to the following nominal marginal cost:

$$MC_t^* = S_t \left(\overline{\Pi}_Y^* P_{OIL,t-1}^*\right)^{\omega^*} \left(P_{Y,t}^*\right)^{1-\omega^*}. \tag{30}$$

Except for the nominal effective exchange rate, S_t, all variables in (30) are expressed in terms of producer currency: $\overline{\Pi}_Y^*$ is the gross steady-state inflation rate in the foreign economy, $P_{OIL,t}^*$ is the global price of oil and $P_{Y,t}^*$ is the global price of foreign intermediate goods. Following Christoffel et al. (2008), we assume an explicit role for the global price of oil, with the parameter ω^* being the oil share in the import basket.[12]

Once differentiated, the imported intermediate goods are supplied as inputs to the final goods firms in monopolistically competitive markets. As in the case of domestically-produced intermediate goods, we employ the Calvo (1983) setup for the consumer-currency pricing of imported goods. In this case, ξ^* is the probability of not receiving an idiosyncratic reoptimization signal, which is followed by the adjustment of prices according to the following price indexation:

$$P_{IM,f^*,t} = \Pi_{IM,t-1}^{\chi^*} \overline{\Pi}_t^{1-\chi^*} P_{IM,f^*,t-1}, \tag{31}$$

where $\Pi_{IM,t} \equiv P_{IM,t}/P_{IM,t-1}$. The parameter χ^* reflects the degree of indexation to past inflation of aggregate imported goods prices.

Upon receiving the idiosyncratic price reoptimization signal, firm f^* revises its (consumer-currency) price so as to maximize profits:

$$\max E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k}^* \left(\xi^*\right)^k \left(P_{IM,f^*,t} IM_{f^*,t} - MC_t^* IM_{f^*,t}\right)/S_t\right]. \tag{32}$$

The components of the discount factor, $\Lambda_{t,t+k}^*$ and $(\xi^*)^k$, are the foreign counterparts of (27).

All reoptimizing firms will choose the same new price, $\widetilde{P}_{IM,t}$, so as to maximize (32)

[12] In our specification, as opposed to Christoffel et al. (2008), global oil prices have a *delayed* effect on the cost of imports.

subject to the indexation scheme (31) and the demand for differentiated goods (equation (40) below). The optimality condition is given by:

$$E_t\left[\sum_{k=0}^{\infty}\Lambda^*_{t,t+k}\left(\xi^*\right)^k\left(\Pi^{\dagger}_{IM,t,t+k}\widetilde{P}_{IM,t}-\varphi^*_{t+k}MC^*_{t+k}\right)IM_{f^*,t+k}/S_{t+k}\right]=0, \qquad (33)$$

where $\Pi^{\dagger}_{IM,t,t+k}\equiv\prod_{s=1}^{k}\left(\Pi^{\chi^*}_{IM,t+s-1}\overline{\Pi}^{1-\chi^*}_{t+s}\right)$ for $k\geq 1$ and $\Pi^{\dagger}_{IM,t,t+k}\equiv 1$ for $k=0$. The variable φ^*_t is the optimal markup of foreign intermediate good firms.

As before, we can derive the following dynamics for the aggregate import price:

$$P_{IM,t}=\left[\left(1-\xi^*\right)\left(\widetilde{P}_{IM,t}\right)^{\frac{1}{1-\varphi^*_t}}+\xi^*\left(\Pi^{\chi^*}_{IM,t-1}\overline{\Pi}^{1-\chi^*}_{IM,t}P_{IM,t-1}\right)^{\frac{1}{1-\varphi^*_t}}\right]^{1-\varphi^*_t}. \qquad (34)$$

3.2.3 Domestic final goods firms

Domestic firms producing final goods are divided into four categories: producers of consumption goods Q^C_t, producers of investment goods Q^I_t, producers of government-consumption goods Q^G_t and producers of exported goods Q^X_t. This section describes the first category, i.e. the producers of final consumption goods. A similar description can be applied to the other categories as well.[13,14]

Technology

The final consumption good is a CES composite of domestically-produced and imported aggregates of intermediate goods (H^C_t and IM^C_t, respectively):

$$Q^C_t=\left(\nu^{\frac{1}{\mu_C}}_{C,t}\left[H^C_t\right]^{1-\frac{1}{\mu_C}}+\left(1-\nu_{C,t}\right)^{\frac{1}{\mu_C}}\left\{\left[1-\Gamma_{IM^C}\left(IM^C_t/Q^C_t;\varepsilon^{IM}_t\right)\right]IM^C_t\right\}^{1-\frac{1}{\mu_C}}\right)^{\frac{\mu_C}{\mu_C-1}}. \qquad (35)$$

[13] With the appropriate changes in parameterization.

[14] Section 3.2.3 below elaborates on some additional steps in the production and marketing of export goods.

The parameter μ_C is the CES between domestic and imported goods while the (time-varying) parameter $\nu_{C,t}$ measures the degree of home bias ($1-\nu_C$ is the steady-state import intensity in the Q_t^C sector).[15]

The aggregates of the domestically-produced and imported intermediate goods are, respectively:

$$H_t^C = \left(\int_0^1 \left(H_{f,t}^C \right)^{\frac{1}{\varphi_t^H}} df \right)^{\varphi_t^H}, \tag{36}$$

and

$$IM_t^C = \left(\int_0^1 \left(IM_{f^*,t}^C \right)^{\frac{1}{\varphi_t^*}} df^* \right)^{\varphi_t^*}. \tag{37}$$

Thus, the optimal markups of the intermediate goods producers, φ_t^H and φ_t^*, are time-varying.

Changing the import intensity in production $\left(IM_t^C/Q_t^C\right)$ involves an adjustment cost:

$$\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\varepsilon_t^{IM}\right) = \frac{\gamma_{IM^C}}{2}\left[\left(\varepsilon_t^{IM}\right)^{-\frac{1}{\gamma_{IM^C}}}\frac{IM_t^C/Q_t^C}{IM_{t-1}^C/Q_{t-1}^C}-1\right]^2, \tag{38}$$

where ε_t^{IM} is an exogenous shock that affects import demand through its effect on the productivity of imported inputs.[16]

Resource allocation and price setting

Treating the prices of intermediate goods as given, optimal allocations *within* the domestically-produced and imported bundles leads to the following demand for inputs:

$$H_{f,t}^C = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H-1}} H_t^C, \tag{39}$$

[15] The time-varying home bias parameters follow AR(1) processes similar to the one in (20).

[16] For ease of exposition, there is some abuse of notation here. The import intensity in the previous period, IM_{t-1}^C/Q_{t-1}^C, is the aggregate one, whereas the intensity in the current period, IM_t^C/Q_t^C, is a choice variable of the individual firm. Hence, the firm's decision ignores the effect of current import intensity on future import productivity.

and

$$IM^C_{f^*,t} = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{-\frac{\varphi^*_t}{\varphi^*_t - 1}} IM^C_t, \tag{40}$$

where the aggregate price index is defined as the price of one unit of the relevant composite, such that $P_{H,t}H^C_t = \int\limits_0^1 P_{H,f,t} \cdot H^C_{f,t} \cdot df$ and $P_{IM,t}IM^C_t = \int\limits_0^1 P_{IM,f^*,t} \cdot IM^C_{f^*,t} \cdot df^*$. Substituting these expressions into (39) and (40) yields, respectively:

$$P_{H,t} = \left(\int\limits_0^1 (P_{H,f,t})^{\frac{1}{1-\varphi^H_t}} \, df\right)^{1-\varphi^H_t}, \tag{41}$$

and

$$P_{IM,t} = \left(\int\limits_0^1 (P_{IM,f^*,t})^{\frac{1}{1-\varphi^*_t}} \, df^*\right)^{1-\varphi^*_t}. \tag{42}$$

In turn, taking the aggregate price indices $P_{H,t}$ and $P_{IM,t}$ as given, an optimal allocation *between* domestically-produced and imported bundles of intermediate goods leads to the following demand functions:

$$H^C_t = \nu_{C,t} \left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\mu_C} Q^C_t, \tag{43}$$

and

$$IM^C_t = (1 - \nu_{C,t}) \left(\frac{P_{IM,t}}{P_{C,t}\Gamma^{\dagger}_{IM^C}\left(IM^C_t / Q^C_t; \varepsilon^{IM}_t\right)}\right)^{-\mu_C} \frac{Q^C_t}{1 - \Gamma_{IM^C}\left(IM^C_t / Q^C_t; \varepsilon^{IM}_t\right)}, \tag{44}$$

where $\Gamma^{\dagger}_{IM^C}\left(IM^C_t / Q^C_t; \varepsilon^{IM}_t\right) = 1 - \Gamma_{IM^C}\left(IM^C_t / Q^C_t; \varepsilon^{IM}_t\right) - \Gamma'_{IM^C}\left(IM^C_t / Q^C_t; \varepsilon^{IM}_t\right) IM^C_t$.

Since final goods firms operate in an environment of perfect competition, they simply charge a price equal to their marginal cost. Thus,

$$P_{C,t} = \left\{ \nu_{C,t} \left[P_{H,t} \right]^{1-\mu_C} + (1-\nu_{C,t}) \left(\frac{P_{IM,t}}{\Gamma^{\dagger}_{IM^C} \left(IM^C_t / Q^C_t ; \varepsilon^{IM}_t \right)} \right)^{1-\mu_C} \right\}^{\frac{1}{1-\mu_C}} . \tag{45}$$

Generalization to other production categories

Deriving the analogous equations for the other sectors (Q^I_t, Q^G_t and Q^X_t) is straightforward and is accomplished by replacing the index C in equations (35) to (45) with I, G or X. The only exception is the price of exported goods which is denoted by $P_{DX,t}$ (the notation $P_{X,t}$ is reserved for the foreign-currency price charged by exporters who buy Q^X_t, brand name it and sell it to foreign retail firms). This is dealt with in the remainder of this section.

3.2.4 Exporters

Final goods, as described in the previous subsection, are supplied under perfect competition. The intermediate goods sector is characterized by monopolistic competition which is essential for the existence of nominal frictions; however, they only induce *domestic* price rigidity. In order to allow for price rigidity in terms of foreign currency as well, i.e. export price rigidity, we further segment the exporting sector into intermediate stages. This subsection focuses on the so-called exporters (see figure 1), who are indexed by $f^x \in [0,1]$. They buy the homogenous export good, Q^X_t, and brand-name it so as to provide a differentiated good, $X_{f^x,t}$. Hence, with an additional sector of differentiated goods in place, monopolistic competition can be imposed on the exporting sector, thus allowing for price rigidity in terms of the foreign currency. Thus, exporter f^x buys the amount $Q^X_{f^X,t}$ of the homogenous export good and brand-name it to become $X_{f^X,t}$ units of differentiated good using a simple production function:

$$X_{f^X,t} = Q^X_{f^X,t} - \psi^X z_t. \tag{46}$$

As in the case of monopolistic producers of domestic intermediate goods, brand naming involves a fixed cost, $\psi^X z_t$.

The Calvo (1983) setup for price rigidity is used here as well. Thus, there is a fixed probability, ξ_X, that an exporter f^x will not get to reoptimize its price, in which case he adjusts his foreign currency price, $P_{X,f^X,t}$, according to the following indexation scheme:

$$P_{X,f^X,t} = (\Pi_{X,t-1})^{\chi_X} \left(\overline{\Pi}_t^*\right)^{(1-\chi_X)} P_{X,f^X,t-1}, \tag{47}$$

where $\Pi_{X,t} \equiv P_{X,t}/P_{X,t-1}$ is the rate of (foreign currency) inflation in the export sector and $\overline{\Pi}_t^*$ is the gross (potentially time-varying) foreign inflation objective.

Upon receiving the idiosyncratic reoptimization signal, firm f^X adjusts prices so as to maximize its discounted dividend flow, while taking into account the price indexation scheme (47) and the demand for its differentiated good (see equation (53) below):

$$\max E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t-1} \xi_X^k \left(S_t P_{X,f^X,t} X_{f^X,t} - MC_t^X X_{f^X,t} \right) \right]. \tag{48}$$

Note that MC_t^X, i.e. exporters' nominal marginal cost in domestic currency, is the price of the homogenous exported good, such that:

$$MC_t^X = P_{DX,t}\ . \tag{49}$$

The optimal foreign currency price, $\widetilde{P}_{X,t}$, is the same for all reoptimizing exporters and satisfies the following optimality condition:

$$E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+1} \xi_X^k \left(\Pi^\dagger_{X,t,t+k} \widetilde{P}_{X,t} - \varphi_{t+k}^X S_t^{-1} MC_{t+k}^X \right) X_{f^X,t+k} \right] = 0, \tag{50}$$

where $\Pi^\dagger_{X,t,t+k} \equiv \prod_{s=1}^{k} \left[\left(\Pi_{X,t+s-1}^{\chi_X}\right)^{\chi_X} \left(\overline{\Pi}^*_{X,t+s}\right)^{1-\chi_X} \right]$ for $k \geq 1$ and $\Pi^\dagger_{X,t,t+k} \equiv 1$ for $k = 1$. φ_t^X is an optimal price-markup shock.

Based on the aggregate price (equation (54) below) and the law of large numbers, the dynamics of the aggregate export price are described by:

$$P_{X,t} = \left[(1-\xi_X) \left(\widetilde{P}_{X,t} \right)^{\frac{1}{1-\varphi_t^X}} + \xi_X \left((\Pi_{X,t-1})^{\chi_X} \left(\overline{\Pi}_{X,t}^{*} \right)^{1-\chi_X} P_{X,t-1} \right)^{\frac{1}{1-\varphi_t^X}} \right]^{1-\varphi_t^X} . \quad (51)$$

3.2.5 Foreign retail firms

Foreign retail firms purchase the differentiated export goods $X_{f^X,t}$, where $f^X \in [0,1]$, and combine them into a homogenous export good, X_t (see figure 1). The homogenous export good, in turn, is a CES aggregate of the differentiated export goods:

$$X_t = \left(\int_0^1 \left(X_{f^X,t} \right)^{\frac{1}{\varphi_t^X}} df^X \right)^{\varphi_t^X} . \quad (52)$$

Taking the price of differentiated goods as exogenously given for foreign retailers, their optimal allocation leads to a standard Dixit and Stiglitz (1977) demand equation:

$$X_{f^X,t} = \left(\frac{P_{X,f^X,t}}{P_{X,t}} \right)^{-\frac{\varphi_t^X}{\varphi_t^X - 1}} X_t, \quad (53)$$

where

$$P_{X,t} = \left(\int_0^1 \left(P_{X,f^X,t} \right)^{\frac{1}{1-\varphi_t^X}} df^X \right)^{1-\varphi_t^X} . \quad (54)$$

Since there are infinitely many foreign retailers who sell a homogenous good, the price of the good is equal to their marginal cost of production, namely $P_{X,t}$. The homogenous export good is combined with other countries' export goods to form a CES aggregate of world trade, WT_t^*. Thus, the demand for Israeli exports is analogous to the demand for imported and domestic intermediate goods in the production of the final goods (see e.g. equation (44)):

$$X_t = \nu_t^* \left(\frac{P_{X,t}}{P_{X,t}^{c,*} \Gamma_X^{\dagger} \left(X_t / WT_t^*; \varepsilon_t^X \right)} \right)^{-\mu^*} \frac{WT_t^*}{1 - \Gamma_X \left(X_t / WT_t^*; \varepsilon_t^X \right)} , \quad (55)$$

where $P^{c,*}_{X,t}$ is the price aggregate of world trade, the parameter μ^* is the price elasticity of exports, and the exogenous process ν^*_t is a country-specific export-demand shock. The function

$$\Gamma_X\left(X_t/WT^*_t;\varepsilon^X_t\right) \equiv \frac{\gamma^*}{2}\left[\left(\varepsilon^X_t\right)^{-\frac{1}{\gamma^*}}\frac{X_t/WT^*_t}{X_{t-1}/WT^*_{t-1}}-1\right]^2 \tag{56}$$

is an adjustment cost associated with changing the composition of world trade,[17] such that:

$$\Gamma^{\dagger}_X\left(X_t/WT^*_t;\varepsilon^X_t\right) \equiv 1-\Gamma_X\left(X_t/WT^*_t;\varepsilon^X_t\right)-\Gamma'_X\left(X_t/WT^*_t;\varepsilon^X_t\right)X_t. \tag{57}$$

3.3 The public sector

3.3.1 The government

The government purchases homogenous final goods (G_t), issues bonds (B_t) and imposes taxes—both distortionary and lump sum. The period-by-period budget constraint faced by the government is given by:

$$\begin{aligned} P_{G,t}G_t+B_t &= \tau^C_tP_{C,t}C_t+\left(\tau^N_t+\tau^{W_h}_t\right)\int_0^1 W_{h,t}N_{h,t}dh+\tau^{W_f}W_tN_t \\ &\quad +\tau^K_t\left[R_{K,t}u_t-\left(\Gamma_u\left(u_t\right)+\delta\right)P_{I,t}\right]K_t+\tau^D_tD_t+T_t+R^{-1}_tB_{t+1}. \end{aligned} \tag{58}$$

We assume exogenous processes for government expenditures and tax rates. Thus, the following AR(1) process is assumed for government spending:

$$g_t=\left(1-\rho_G\right)g+\rho_Gg_{t-1}+\eta^G_t, \tag{59}$$

where government spending is stationarized by productivity so that $g_t\equiv G_t/z_t$, and the VAT rate is essentially assumed to be a random walk process:

[17] With a similar abuse of notation to that in the case of the adjustment costs faced by domestic final goods firms (38).

$$\tau_t^C = (1-0.99)\,\tau^C + 0.99\tau_{t-1}^C + \eta_t^{\tau^C}. \tag{60}$$

The other tax rates—τ_t^N, $\tau_t^{W_h}$, τ^{W_f}, τ_t^K and τ_t^D—are assumed to be constant.

We assume that the allocation between lump-sum taxes (T_t) and the issue of debt (B_{t+1}) to finance government spending (in order for the budget constraint (58) to be satisfied), is determined by the following rule:

$$s_{T,t} = \phi_B\left(s_{B,t+1} - s_B\right). \tag{61}$$

The variables $s_{T,t} \equiv \frac{T_t}{P_{Y,t}Y_t}$ and $s_{B,t+1} \equiv \frac{B_{t+1}}{P_{Y,t}Y_t}$ are, respectively, lump-sum taxes and the outstanding government debt, both in terms of their share in GDP. Note that since distortionary taxes are exogenous, "Ricardian equivalence" holds and the (somewhat arbitrary) specification of the financing rule (61) does not affect the rest of the model. Also note that (61) ensures the convergence of government debt to its steady-state value in the long run ($E_t\left[s_{B,t+\infty}\right] \rightarrow s_B$).

3.3.2 The central bank

The central bank sets the nominal interest rate, R_t, using an inflation-expectation-based rule. We follow the literature by generalizing a Taylor (1993) type rule, with standard modifications such as those in Christoffel et al. (2008) and Adolfson et al. (2007), among others. We also add the forward interest rate to the policy rule, and include a response to a *four-quarter* inflation rate and to nominal depreciation. In terms of log-linear deviations from the deterministic steady state, the policy rule takes the following form:

$$\begin{aligned}\hat{r}_t \;&=\; (1-\phi_R)\left[\widehat{rr}_t^{fwd} + \widehat{\overline{\pi}}_t + \phi_\Pi\left(\hat{\pi}_t^{CB} - \widehat{\overline{\pi}}_t\right) + \phi_y \hat{y}_t^{GAP} + \phi_{\Delta S}\Delta S_t\right] \\ &\quad +\phi_R \hat{r}_{t-1} + \eta_t^R.\end{aligned} \tag{62}$$

Thus, policy reacts to deviations of (expected) inflation from the inflation target $\left(\hat{\pi}_t^{CB} - \widehat{\overline{\pi}}_t\right)$, deviations of output from a technology-driven trend $\left(\hat{y}_t^{GAP} \equiv \log\frac{Y_t}{Z_t\varepsilon_t} - \log y\right)$ and nominal depreciation $\left(\Delta S_t \equiv \Delta\hat{s}_t + \hat{\pi}_{Y,t} - \hat{\pi}_{Y,t}^*\right)$.

The variable $\widehat{rr}_t^{fwd}$ is the forward real interest rate, i.e. the average of the real rates expected to prevail 5 to 10 years ahead:

$$\widehat{rr}_t^{fwd} = \frac{1}{20} E_t \left[\widehat{ri}_{t+21} + \widehat{ri}_{t+22} + \cdots + \widehat{ri}_{t+39} + \widehat{ri}_{t+40} \right], \tag{63}$$

where $\widehat{ri}_t \equiv \widehat{r}_t - E_t \widehat{\pi}_{C,t+1}$ is the (log linearized) real interest rate. $\widehat{rr}_t^{fwd}$ is governed by ε_t^{DRP}, the domestic, and highly inertial, risk-premium shock.[18]

In order to account for the disinflation process characterizing the first half of the sample period, a time-varying inflation target, $\widehat{\overline{\pi}}_t$, is introduced which essentially follows a random-walk process:

$$\widehat{\overline{\pi}}_t = 0.99 \widehat{\overline{\pi}}_{t-1} + \eta_t^{\overline{\Pi}}. \tag{64}$$

Empirical as well as theoretical findings by Argov and Elkayam (2010) motivated the direct response of interest rate policy to nominal depreciation and the response to both historical and expected inflation. Thus, the inflation measure to which the central bank reacts is defined as:

$$\hat{\pi}_t^{CB} = E_t \left[\hat{\pi}_{C,t-2} + \hat{\pi}_{C,t-1} + \hat{\pi}_{C,t} + \hat{\pi}_{C,t+1} \right]. \tag{65}$$

Finally, the policy shock, η_t^R, follows a white noise process.

3.4 Net foreign assets and the current account

Let

$$TB_t = P_{X,t} S_t X_t - P_{IM,t} IM_t \tag{66}$$

be the trade balance and

$$CA_t = TB_t + FTR_t \tag{67}$$

[18] See section (3.1) and footnote 7.

the current account, where FTR_t denotes exogenous foreign transfers. It then follows that the net foreign assets of the domestic economy evolve according to the following law of motion:

$$(R_t^*)^{-1} B_{t+1}^* = B_t^* + \frac{CA_t}{S_t} \quad . \tag{68}$$

We assume the following AR(1) process for $s_{FTR,t} = \frac{FTR_t}{P_{Y,t}Y_t}$, i.e. foreign transfers expressed as a share of nominal GDP:

$$s_{FTR,t} = (1 - \rho_{FTR})\, s_{FTR} + \rho_{FTR} \cdot s_{FTR,t-1} + \eta_t^{FTR}. \tag{69}$$

3.5 Market-clearing conditions

3.5.1 Clearing of the labor market

In order to satisfy labor-market equilibrium, the demand of all intermediate goods firms for differentiated labor services is met by the supply provided by households. Thus,

$$N_{h,t} = \int_0^1 N_{f,t}^h df. \tag{70}$$

Aggregating over the continuum of all households $h \in [0,1]$:

$$\int_0^1 N_{h,t} dh = \int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W - 1}} \cdot N_t \cdot dh = \check{s}_{W,t} \cdot N_t, \tag{71}$$

where $\check{s}_{W,t} \equiv \int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W - 1}} \cdot dh$ is a measure of wage dispersion and $N_t \equiv \int_0^1 N_{f,t} \cdot df = \int_0^1 \left[\left(\int_0^1 \left(N_{f,t}^h\right)^{\frac{1}{\varphi_t^W}} dh\right)^{\varphi_t^W}\right] df$ is the production relevant aggregate of differentiated labor services. Equation (71) links the simple sum and the production aggregation of labor services.

The economy's total payroll is

$$\int_0^1 N_{h,t} \cdot W_{h,t} \cdot dh = N_t \int_0^1 W_{h,t} \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W - 1}} dh = W_t N_t. \tag{72}$$

The first equality follows from the aggregate demand for labor services (71) and the second is based on the wage aggregate (22).

3.5.2 Clearing of the capital market

To satisfy capital market equilibrium, firms' demand for capital services $\left(K_t^s \equiv \int_0^1 K_{f,t}^s df\right)$ is met by households' utilized capital stock $\left(u_t K_t \equiv \int_0^1 u_{h,t} K_{h,t} dh\right)$, such that:

$$u_t K_t = K_t^s. \tag{73}$$

3.5.3 Intermediate-goods market clearing

Domestic intermediate goods

The supply of differentiated domestic goods by the $f'th$ firm, $H_{f,t}^s$, meets the demand in all sectors (C,I,G and X), such that:

$$H_{f,t}^s = H_{f,t}^C + H_{f,t}^I + H_{f,t}^G + H_{f,t}^X. \tag{74}$$

Using (39) and aggregating over f, the continuum of firms:

$$\int_0^1 H_{f,t}^s \cdot df = \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} \cdot H_t \cdot df,$$

where $H_t \equiv H_t^C + H_t^I + H_t^G + H_t^X$ collects all production-relevant aggregates of differentiated intermediate domestic goods (as in, for example, equation 36). By defining the simple integral on the left-hand side as $H_t^s \equiv \int_0^1 H_{f,t}^s \cdot df$ and substituting $\check{s}_{H,t} \equiv \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} df$ for the measure of price dispersion, we obtain a straightforward expression linking the sum of demands and the production aggregate of domestic intermediate goods:

$$H_t^s = \check{s}_{H,t} H_t. \tag{75}$$

With regard to the market-clearing price, the aggregate nominal expenditure on domestic intermediate goods is given by:

$$\int_0^1 P_{H,f,t} H_{f,t}^s df = H_t \int_0^1 P_{H,f,t} \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} df = P_{H,t} H_t \ . \tag{76}$$

The first equality makes use of the market-clearing condition (74) and the demand equation (39) for domestic intermediate goods produced by firm f. The second equality makes use of the price index equation (41).

Imported intermediate goods

Market-clearing conditions for imported intermediate goods are derived analogously to those for domestic intermediate goods. Thus, defining $\check{s}_{IM,t} \equiv \int_0^1 \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^* - 1}} df^*$ and $IM_t^s \equiv \int_0^1 IM_{f^*,t} df^*$, we obtain the following equations, which are analogous to (74-76):

$$IM_{f^*,t} = IM_{f^*,t}^C + IM_{f^*,t}^I + IM_{f^*,t}^G + IM_{f^*,t}^X, \tag{77}$$

$$IM_t^s = \check{s}_{IM,t} IM_t \tag{78}$$

and

$$\int_0^1 P_{IM,f^*,t} IM_{f^*,t} df^* = P_{IM,t} IM_t, \tag{79}$$

where $IM_t \equiv IM_t^C + IM_t^I + IM_t^G + IM_t^X$.

3.5.4 Clearing of the final goods markets

Clearing conditions in the competitive domestic final goods market are as follows:

$$Q_t^C = C_t \ , \tag{80}$$

$$Q_t^I = I_t + \Gamma_u \left(u_{h,t}\right) \cdot K_t + \Delta INV_t \tag{81}$$

and

$$Q_t^G = G_t \ . \tag{82}$$

Market clearing in the export sector implies:

$$Q_t^X = \int_0^1 Q_{f^X,t} \cdot df^X = \int_0^1 X_{f^X,t} \cdot df^X + \int_0^1 \psi^X \cdot z_t \cdot df^X,$$

where the first equality is the market-clearing condition in the domestic homogenous export goods market and the second equality makes use of the differentiated export goods production function (46).

Using the demand faced by exporting firms (53) and defining the degree of price dispersion in the export sector $\check{s}_{X,t} \equiv \int_0^1 \left(\frac{P_{X,f^X,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{\varphi_t^X - 1}} \cdot df^X$ we obtain the following link between the production of exported goods, Q_t^X, and the utility-based export aggregate, X_t :

$$Q_t^X = \check{s}_{X,t} \cdot X_t + \psi^X \cdot z_t. \tag{83}$$

3.5.5 Aggregate resource constraints

Let $P_{Y,t}Y_t$ be nominal GDP, i.e. the aggregate added value of the domestic economy. Since the added value of the perfectly competitive firms is nil, it follows that:

$$P_{Y,t}Y_t = P_{H,t}H_t^s + S_t P_{X,t} X_t - P_{DX,t} Q_t^X. \tag{84}$$

Using the zero-profit conditions for competitive final-goods firms and taking into account the market-clearing conditions for intermediate and final goods, we obtain the aggregate nominal resource constraint:

$$\begin{aligned} P_{Y,t}Y_t \;=\; & P_{C,t}C_t + P_{I,t}\left(I_t + \Gamma_u\left(u_{h,t}\right)K_t + \Delta INV_t\right) + P_{G,t}G_t + S_t P_{X,t}X_t \\ & - P_{IM,t}\left(IM_t^C \frac{1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)}{\Gamma_{IM^C}^{\dagger}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)} + IM_t^I \frac{1-\Gamma_{IM^I}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)}{\Gamma_{IM^I}^{\dagger}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)}\right) \\ & - P_{IM,t}\left(IM_t^G \frac{1-\Gamma_{IM^G}\left(IM_t^G/Q_t^G;\epsilon_t^{IM}\right)}{\Gamma_{IM^G}^{\dagger}\left(IM_t^G/Q_t^G;\epsilon_t^{IM}\right)} + IM_t^X \frac{1-\Gamma_{IM^X}\left(IM_t^X/Q_t^X;\epsilon_t^{IM}\right)}{\Gamma_{IM^X}^{\dagger}\left(IM_t^X/Q_t^X;\epsilon_t^{IM}\right)}\right). \end{aligned} \tag{85}$$

The constraint can also be expressed in terms of market prices by adding VAT, which will be useful when taking the model to the data:

$$\begin{aligned} P_{Y,t}^M Y_t \;=\; & \left(1+\tau_t^C\right)P_{C,t}C_t + P_{I,t}\left(I_t + \Gamma_u\left(u_{h,t}\right)K_t + \Delta INV_t\right) + P_{G,t}G_t + S_t P_{X,t}X_t \\ & - P_{IM,t}\left(IM_t^C \frac{1-\Gamma_{IM^C}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)}{\Gamma_{IM^C}^{\dagger}\left(IM_t^C/Q_t^C;\epsilon_t^{IM}\right)} + IM_t^I \frac{1-\Gamma_{IM^I}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)}{\Gamma_{IM^I}^{\dagger}\left(IM_t^I/Q_t^I;\epsilon_t^{IM}\right)}\right) \\ & - P_{IM,t}\left(IM_t^G \frac{1-\Gamma_{IM^G}\left(IM_t^G/Q_t^G;\epsilon_t^{IM}\right)}{\Gamma_{IM^G}^{\dagger}\left(IM_t^G/Q_t^G;\epsilon_t^{IM}\right)} + IM_t^X \frac{1-\Gamma_{IM^X}\left(IM_t^X/Q_t^X;\epsilon_t^{IM}\right)}{\Gamma_{IM^X}^{\dagger}\left(IM_t^X/Q_t^X;\epsilon_t^{IM}\right)}\right). \end{aligned} \tag{86}$$

We define real output as the output produced by the domestic intermediate-goods firms, i.e. using the economy's factors of production (labor and capital):[19]

[19] The definition of real output, namely the partition of nominal output into real output and the GDP deflator, is needed when taking the model to the data, i.e. in relating the model's variables to the corresponding observable variables. Also, the output gap appears in the monetary policy rule (62). Note that our definition of real output excludes the exporters' monopolistic profits, although it is included in the definition of nominal output (84).

$$Y_t = H_t^s. \tag{87}$$

3.5.6 The share of profits

The profits from the production of domestic intermediate goods and exports are given by:

$$\begin{aligned} D_t &\equiv \int_0^1 D_{H,f,t} df + \int_0^1 D_{X,f^X,t} df^X = \\ &= \int_0^1 \left[P_{H,f,t} H_{f,t}^s - MC_t \left(H_{f,t} + \psi z_t\right)\right] df + \int_0^1 \left[S_t P_{X,f^X,t} X_{f^X,t} - MC_t^X \left(X_{f^X,t} + \psi^X z_t\right)\right] df^X \\ &= P_{H,t} H_t^s - MC_t \left(\check{s}_{H,t} H_t + \psi z_t\right) + S_t P_{X,t} X_t - P_{DX,t} \left(\check{s}_{X,t} X_t + \psi^X z_t\right). \end{aligned} \tag{88}$$

Using (84), we can express the profits in terms of their share in nominal GDP:

$$s_{D,t} \equiv \frac{D_t}{P_{Y,t} Y_t} = \frac{P_{Y,t} Y_t - MC_t \left(\check{s}_{H,t} H_t + \psi z_t\right)}{P_{Y,t} Y_t} = 1 - \frac{MC_t}{P_{Y,t}} \frac{\left(\check{s}_{H,t} H_t + \psi z_t\right)}{Y_t}.$$

3.6 The foreign economy

The domestic economy is influenced by global conditions through five foreign variables: the interest rate (R_t^*), intermediate good prices ($P_{Y,t}^*$), oil prices ($P_{OIL,t}^*$), prices of competing exporters ($P_{X,t}^{c,*}$) and world trade (WT_t^*).

There are various approaches to modeling the foreign economy, which is exogenous to the domestic economy. Christoffel et al. (2008) and Adolfson et al. (2007) use Structural VAR while Argov et al. (2007) employ univariate auto-regressive equations to characterize the dynamics of foreign variables. Experimentation with such approaches produced unsatisfactory impulse responses for foreign shocks. Therefore, we chose to specify a simple closed-economy, New-Keynesian-style model for the foreign economy, which is presented below in its log-linearized form. Small hatted letters denote log deviations from a deterministic steady state, and *epsilons* denote exogenous shocks.

In order to specify foreign output, $\hat{y}_t^*$, we use a hybrid (i.e. both forward- and backward-looking) investment-saving equation:

$$\hat{y}_t^* = c_{y^*,+} E_t\left[\hat{y}_{t+1}^*\right] + (1 - c_{y^*,+})_t \, \hat{y}_{t-1}^* - c_{y^*,r} \cdot 4 \cdot \left(\hat{r}_t^* - E_t\left[\hat{\pi}_{Y,t+1}^*\right] - \widehat{rr}_t^{*,fwd}\right) + \varepsilon_t^{Y^*}. \quad (89)$$

This is a relatively standard specification, except for the use of $\widehat{rr}_t^{*,fwd}$ as a proxy for the foreign "natural" interest rate. The observable forward nominal interest rate is used to identify it within the data. Based on the behavior of this variable, as well as that of short-run nominal interest rates worldwide, we assume that it follows a nearly random walk process:

$$\widehat{rr}_t^{*,fwd} = 0.99 \cdot \widehat{rr}_{t-1}^{*,fwd} + \varepsilon_t^{*,fwd}. \quad (90)$$

In order to link global output ($\hat{y}_t^*$), which is specified by (89), and world trade ($\widehat{wt}_t^*$), which drives domestic exports in equation (55), we assume the following process:

$$\widehat{wt}_t^* = c_{wt,y}\hat{y}_t^* + c_{wt,y\_lag}\hat{y}_{t-1}^* + c_{wt,-}\widehat{wt}_{t-1}^* + \varepsilon_t^{WT^*}. \quad (91)$$

World inflation, $\hat{\pi}_{Y,t}^*$, is subject to a hybrid New Keynesian Phillips curve:

$$\begin{aligned} 4 \cdot \hat{\pi}_{Y,t}^* \;=\; & c_{\pi^*,+} \cdot 4 \cdot E_t\left[\hat{\pi}_{Y,t+1}^*\right] + (1 - c_{\pi^*,+}) \cdot 4 \cdot \hat{\pi}_{Y,t-1}^* \\ & + c_{\pi^*,y}\frac{\hat{y}_t^* + \hat{y}_{t-1}^*}{2} + c_{\pi^*,OIL}\hat{p}_{OIL,t}^* + c_{\pi^*,\Delta OIL}\left(\hat{p}_{OIL,t}^* - \hat{p}_{OIL,t-2}^*\right) + \varepsilon_t^{\Pi^*}, \end{aligned} \quad (92)$$

where the relative price of oil, $p_{OIL,t}^* \equiv P_{OIL,t}^*/P_{Y,t}^*$, follows an AR(2) process:

$$\hat{p}_{OIL,t}^* = c_{oil,-}\hat{p}_{OIL,t-1}^* + c_{oil,\Delta}\left(\hat{p}_{OIL,t-1}^* - \hat{p}_{OIL,t-2}^*\right) + \varepsilon_t^{OIL}. \quad (93)$$

The foreign economy model is closed using an extended Taylor (1993)-type rule:

$$\begin{aligned} 4 \cdot \hat{r}_t^* \;=\; & (1 - c_{r^*,-})\Bigg[4 \cdot \left(\widehat{rr}_t^{*,fwd} + \widehat{\overline{\pi}}_t^*\right) \\ & + c_{r^*,\pi} 4 \cdot \left(\frac{\hat{\pi}_{Y,t-1}^* + \hat{\pi}_{Y,t}^* + \hat{\pi}_{Y,t+1}^* + \hat{\pi}_{Y,t+2}^* + \hat{\pi}_{Y,t+3}^*}{5} - \widehat{\overline{\pi}}_t^*\right) \\ & + c_{r^*,y}\hat{y}_t^*\Bigg] + c_{r^*,-} \cdot 4 \cdot \hat{r}_{t-1}^* + \varepsilon_t^{R^*}. \end{aligned} \quad (94)$$

Finally, we assume away variations in the relative prices of the exporters' competitors, such that $p^{c,*}_{X,t} \equiv P^{c,*}_{X,t}/P^{*}_{Y,t} = 1$.

4 Estimation of the model

The model was estimated using Israeli data for the period 1992:Q1 to 2009:Q4, with the first 12 quarters used only to initialize the Kalman filter algorithm.

Section 4.1 describes the data while section 4.2 describes the model-consistent approach used to filter the observed data. Filtering was required in order to estimate a cyclical model with balanced growth using data characterized by numerous structural transitions and breaks.

We estimated a log-linearized version of the model using the Bayesian approach, which became a common practice following Smets and Wouters (2003). Section 4.3 provides a brief overview of the estimation methodology and section 4.4 describes the calibration of some of the parameters, the shocks employed and the prior distributions of the estimated parameters. Section 4.5 describes the estimation results. Finally, section 4.6 presents a sensitivity analysis.

4.1 Data

Twenty-four macroeconomic time series were employed in the estimation. Most are expressed in terms of their log difference, i.e. $\Delta X_t \equiv \log\left(\frac{X_t}{X_{t-1}}\right)$, except for the interest rates, VAT and the current account (which is expressed in terms of its share in GDP). Hours worked, employment and domestic national accounts data are expressed in per-capita terms. Most of the variables had to be adjusted for seasonality, with the exception of interest rates, the exchange rate, tax rates and the price of oil. Nominal variables including rates of inflation, changes in the exchange rate, nominal wage inflation and interest rates were detrended using the inflation target. Following is the full set of observable variables:

- GDP (ΔY_t)
- Private consumption (ΔC_t)
- Fixed capital investment (ΔI_t)
- Government consumption (ΔG_t)
- Exports (ΔX_t)
- Imports (ΔIM_t)
- GDP deflator ($\Delta P^M_{Y,t}$)
- Export deflator ($\Delta P^{NIS}_{X,t} = \Delta \left(S_t P_{X,t} \right)$)
- Current account ($s_{CA,t} = CA_t / P^M_{Y,t} Y_t$)
- CPI ($\Delta P_{C,t}$)
- Inflation target (annualized) ($4 \cdot \overline{\pi}_t$)
- Nominal exchange rate (ΔS_t)
- Nominal hourly wage (ΔW_t)
- Hours worked (ΔN_t)
- Employment (ΔEM_t)
- Bank of Israel key interest rate (r^{OB}_t)
- 5-10 year *fwd* real rate ($rr^{fwd,OB}_t$)
- VAT rate (τ^C_t)
- G4 nominal interest rate ($r^{*,OB}_t$)
- G4 CPI ($\Delta P^*_{Y,t}$)
- G4 GDP (ΔY^*_t)
- OECD imports (ΔWT^*_t)
- 5-10 year *fwd* G4 nominal interest rate ($r^{*,fwd,OB}_t$)
- Price of oil ($\Delta P^*_{OIL,t}$)

In the theoretical model, the quantity of labor is measured by per capita hours worked, N_t. In other words, the labor market in the model does not distinguish between the intensive and extensive margins. However, employment (EM_t) may contain useful information on the degree to which the number of hours worked deviates from some unobserved equilibrium level. Therefore, in order to fully utilize the information inherent in the employment data, we use a semi-theoretical equation linking hours worked to employment, as suggested by

Smets and Wouters (2003) and Christoffel et al. (2008):

$$\widehat{EM}_t = \frac{\beta}{1+\beta\chi_{EM}} E_t\left[\widehat{EM}_{t+1}\right] + \frac{\chi_{EM}}{1+\beta\chi_{EM}} \widehat{EM}_{t-1} + \frac{(1-\beta\xi_{EM})(1-\xi_{EM})}{\xi_{EM}(1+\beta\chi_{EM})}\left(\widehat{N}_t - \widehat{EM}_t\right) + \varepsilon_t^{EM}. \tag{95}$$

A hat denotes a log-deviation from steady state. Equation (95) is based on the assumption that employment adjusts only gradually, while hours worked are more flexible. The parameter ξ_{EM} (which is analogous to the Calvo parameter) is negatively related to the sensitivity of $\widehat{EM}_t$ to $\widehat{N}_t$, while the parameter χ_{EM} generates persistence in the dynamics of employment.[20] The shock, ε_t^{EM}, is neither structural nor is there any feedback from it to the rest of the model. Equation (95) will also be useful in forecasting employment on the basis of the predicted dynamics of hours worked.

4.2 Filtering the observed data

The sample period is characterized by numerous structural transitions and breaks: a disinflation process, capital flow deregulation, exchange rate liberalization, changes in the exchange rate passthroughs, a large wave of immigration, changes in the composition of the export sector, a transition from chronic deficits in the current account to surpluses, a reduction in the government spending-to-output ratio and an increase in the degree of openness, among others. Detailed surveys of the period can be found in Elkayam (2003), Binyamini et al. (2008) and Eckstein and Ramot-Nyska (2008), among others.

As a result, real variables grow at different rates than that of overall output during the sample period (see figure 2). Some of the trends can be partially explained intuitively. For example: the increasing import and export shares reflect the globalization of the Israeli economy; a catch-up process, which explains the convergence of the composition of consumption to that characterizing the G4, can also explain the non-cyclical component of the

[20] Note that the persistence parameter, χ_{EM}, does not appear in Smets and Wouters (2003) or in Christoffel et al. (2008). It has been added here so as to loosen the connection between hours worked and employment and to allow for more general dynamics.

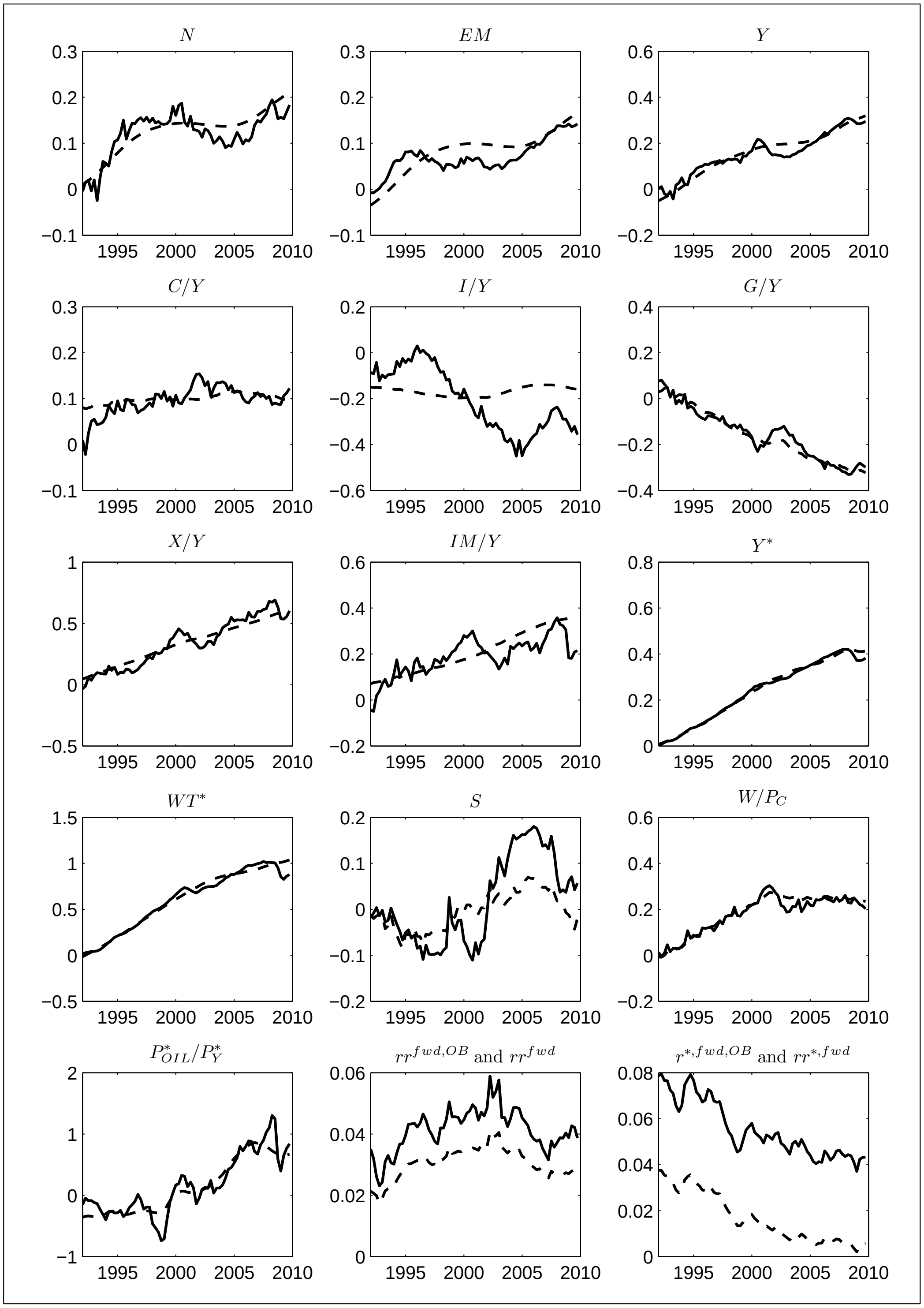
real exchange rate; changes in market structure partially explain why real wages do not trend as much as per-capita output; immigration patterns partially explain the behavior of the share of investment; the increasing labor market participation rate partially explains the non-stationary behavior of per-capita working hours; practices in the market for rental housing explain the weakening of the exchange rate to CPI passthrough.

The model, which is characterized by balanced growth paths determined by the productivity growth rate, g_z, abstracts from all these non-cyclical issues. There are various ways to deal with imbalanced growth rates in the data. For example, if the model is simply estimated using the raw data, the model's transitory shocks are forced to explain the imbalanced growth paths. However, to the extent that these shocks are meant to reflect business cycle dynamics, using the raw data is not desirable. A commonly-used alternative is to remove excess trends using a univariate approach of prefiltering prior to the estimation procedure, as in Christoffel et al. (2008) among others.

We employ a model-consistent filtering approach, along the lines of the "additive hybrid models" described by Schorfheide (2011) and Canova (2009).[21] In this approach, the imbalanced growth paths are extracted simultaneously with the estimation of the model's parameters and shocks. In other words, the raw data is smoothed so as to remove the components that are viewed as being neither cyclical nor balanced trends. A notable advantage of this approach is that it is multivariate, i.e. it exploits the information contained in all the observable variables simultaneously in order to identify the non-cyclical components of each series under consideration. In other words, we use the Kalman-smoother algorithm to remove only those parts of the data that cannot be well-explained by the theoretical model's cyclical behavior. Such a model-consistent approach to filtering the data avoids any pre-filtering and therefore any loss of relevant information contained in the observed data. Hence, if a co-movement inherent in the data can be attributed to some of the structural business-cycle shocks, this information is utilized during the shock extraction.

[21] The employment equation (95) is also a form of such an "additive hybrid model".

Figure 2: Observable Trending Variables (solid line) and Trends (dashed line).



Note: Trends are determined by the stochastic growth rate, g_z, and additive components discussed in section 4.2. Interest rates are annualized. Solid line: observable trending variable. Dashed line: trend.

Observation equations, which connect the structural model to the data, were therefore introduced. Since shocks outnumber observed variables, the Kalman filter is employed during the model estimation. A typical observation equation for a real variable (hours worked, GDP, etc.) takes the following form:

$$\Delta X_t = \hat{x}_t - \hat{x}_{t-1} + J^g \cdot (\widehat{g}_{z,t} + g_z) + J^N \cdot GR_t^N + EX_t \quad ,$$

where X_t is an observed variable, $\hat{x}_t$ is its model-consistent counterpart (in log-deviations from the steady state) and Δ denotes the log-difference operator. The selection indicators J^g and J^N take the value of zero or one. The variable $\widehat{g}_{z,t}$ is the growth rate of the labor-augmenting productivity shock whose process is specified by equation (18) while the variable GR_t^N is the *unobserved* trend in the growth rate of hours worked (which is specified as an AR process). Finally, EX_t is an idiosyncratic trend shock, which is characterized by an AR(1) process. This specification decomposes the component shared by some trending variables into two unobserved components: the technology growth rate from the theoretical model and the trend in the growth rate of hours worked, which is filtered from the hours worked and employment data. Thus, not only is this block helpful in a model-consistent filtering of the data, it is also useful in identifying the latent component $\widehat{g}_{z,t}$.

In addition, since the observed interest rates, both domestic and foreign, do not appear to satisfy stationarity, it proved useful to treat them in a similar manner as trending variables. Thus, there are also two equations connecting the forward interest rates (domestic and foreign) and certain unobserved time-varying term premiums, to their observed (market based) counterparts.

Appendix B provides a detailed description of the observation equations that connect the model to each of the observed variables mentioned in subsection 4.1.

4.3 Methodology

The model is estimated using full-information likelihood-based Bayesian methods [see An and Schorfheide (2007)]. This follows the approach commonly found in the literature, which makes it possible to combine priors on the parameters with the information in the data, as represented by the likelihood of the model. Smets and Wouters (2003), Christoffel et al. (2008) and Adolfson et al. (2007) are all examples of medium scale policy oriented DSGE models estimated by Bayesian methods. The estimation was performed using the Dynare 4 Matlab-based application [see Juillard (1996) and Adjemian et al. (2011)].

In Bayesian econometrics, the posterior distribution of a set of parameters θ_i, which is based on the observed data y and the model at hand M_i, is given by:

$$p\left(\theta_i \left| y, M_i\right.\right) = \frac{p\left(y \left| \theta_i, M_i\right.\right) p\left(\theta_i \left| M_i\right.\right)}{p\left(y \left| M_i\right.\right)}, \tag{96}$$

where $p\left(y \left| \theta_i, M_i\right.\right)$ is the likelihood function that can be computed using the Kalman filter algorithm and $p\left(\theta_i \left| M_i\right.\right)$ is the prior distribution reflecting the researcher's a-priori (i.e. prior to observing the data) assessments regarding θ_i. Since we are only interested in learning about θ_i, we can drop the term $p\left(y \left| M_i\right.\right)$ and focus on the kernel of the distribution:

$$p\left(\theta_i \left| y, M_i\right.\right) \propto p\left(y \left| \theta_i, M_i\right.\right) p\left(\theta_i \left| M_i\right.\right) = K\left(\theta_i \left| y, M_i\right.\right). \tag{97}$$

In general, it is impossible to calculate the distribution function $p\left(\theta_i \left| y, M_i\right.\right)$ or its various moments analytically. Hence, we first use a numerical optimizer to find its mode, and in the second stage the posterior distribution is simulated using a Monte Carlo sampling algorithm. The random-walk Metropolis-Hasting algorithm is used to generate draws from the posterior distribution in order to calculate the mean and selected intervals of the distribution.

The Bayesian methodology also makes it possible to compare model probabilities (between say model i and j) using the Bayes factor $\left(BF_{ij}\right)$, which compares their marginal likelihoods $p\left(M_i \left| y\right.\right)$, while assuming the same prior probabilities for each. The Bayes factor is given by:

$$BF_{ij} = \frac{p(M_i|y)}{p(M_j|y)} = \frac{p(y|M_i)}{p(y|M_j)} = \frac{\int p(y|\theta_i, M_i)\, p(\theta_i|M_i)\, d\theta_i}{\int p(y|\theta_j, M_j)\, p(\theta_j|M_j)\, d\theta_j} = \frac{\int K(\theta_i|y, M_i)\, d\theta_i}{\int K(\theta_j|y, M_j)\, d\theta_j}.$$

The second equality uses Bayes' rule (with equal prior model probabilities); the third equality integrates out the models' parameters; and the last equality uses the definition of the kernel in (97).

4.4 Calibrated parameters, shocks and prior distributions

The model's parameters are divided into two groups: (1) parameters that govern the steady-state solution of the model, which are calibrated so that the steady state is consistent with presumed long-run great ratios (shares in GDP), input weights in production or (imbalanced) growth rates; (2) parameters that govern only the dynamics of the system, which are in general estimated. Subsection 4.4.1 discusses the calibration. The structural shocks used in the estimation are listed in subsection 4.4.2 and the choice of the prior distributions for the estimated parameters is discussed in subsection 4.4.3.

4.4.1 Calibrated parameters

Table 1 presents the calibration of the structural parameters. As noted above, the guiding principle for the calibration was to set the model's steady-state values and ratios to those observed in the data over long horizons or those that are viewed to be the convergence values for the economy. The parameter values are set to obtain the following great ratios in the steady state: private consumption - 55%, fixed capital investment - 21%, inventory investment (Δinv) - 1%, government consumption (s_G) - 26%, exports - 39% and imports - 42%. The trade balance deficit in the steady state is facilitated by a ratio of foreign transfers to GDP (s_{FTR}) of 3%. These are approximately the average ratios observed in the data or those we expect the economy to converge to. The steady-state inflation objective ($\overline{\Pi}$)

was calibrated at an annual rate of 2%, which is located in the middle of the inflation target range during the post-disinflation era. The labor productivity growth parameter (g_z) was set so as to reflect a growth rate of 1.0% in annual terms, which is approximately the sample average. The weight of capital in the production function (α) was calibrated to obtain a wage bill share in GDP of 60%.[22] The depreciation rate (δ) was calibrated to 2% per quarter, which is approximately the long-run average wedge between the gross and net returns on capital. The discount factor β was calibrated so that the steady-state real interest rate equals 2.9%.

For the calibration of certain parameter values, we followed what is common practice in the literature. Thus, we calibrated the inverse of the labor supply elasticity (ζ) to 2.0. The steady-state markups (φ^W, φ^H, φ^*, φ^X) were set at 30% in the wage, domestic and import sectors; in the export sector a smaller markup was chosen (10%) since the monopolistic exporters' price of inputs (P_t^{DX}) is already marked up over marginal cost due to the domestic and import price markups. The steady-state elasticities of substitution between domestic and imported intermediate goods in the private consumption, investment and export sectors (μ^C, μ^I, μ^X) were calibrated to 1.1, which is lower than the values commonly used in the literature[23] but higher than the estimate of 0.4 found for the Israeli economy by Friedman and Lavi (2007). We assumed a very low elasticity (0.2) of substitution in government consumption (μ^G), given that the government's main expenditure is public sector wages, which cannot be substituted for. The foreign elasticity of substitution between imports from different countries (μ^*) was set to 1.5, a value commonly used in the literature. The home bias parameters (ν^C, ν^I, ν^G, ν^X) were calibrated according to the following imports intensities in the steady state: 31% in private consumption, 42% in investment, 5% in government consumption and 32% in exports.[24]

[22] Although we adopt a 60% wage bill share, $(1-\alpha) = 0.67$ due to the cost of working capital, which we do not include in the wage bill, and the adjustment to market prices.

[23] Christoffel et al. (2008) set a prior of 1.5 while Adolfson et al. (2007) calibrate this parameter to 5.0.

[24] Note the significant share of imports in the production of exports. As mentioned above, Christoffel et al. (2008), Adolfson et al. (2007) and Adolfson et al. (2008) assume that exports are comprised of added

The following tax rates were imposed in order to finance government consumption in the steady state: 16% for the consumption tax (τ^C), which is levied on 78% of the consumption basket (w_{τ^C}),[25] 28% for the labor income tax (τ^N), 9% for the payroll tax paid by households (τ^{W_h}) and 7% for the payroll tax paid by firms (τ^{W_f}). In addition, we calibrated the capital income tax (τ^K) to 50%, which is much higher than the actual tax rates on profits or on capital gains, in order to fine tune the steady-state investment-to-GDP ratio . The share of government transfers in GDP (s_{TR}) was calibrated to ensure that the government's budget is balanced in the steady state (which does not affect the linearized model).

The parameters in the export demand function were set as follows: the export competitors' relative price (p_X^C) and the steady-state relative level of foreign technology ($\widetilde{z}$) were normalized to 1.0 and the steady-state weight of Israel's exports in world trade was calibrated to 0.5%. (These three parameters do not affect the linearized model).

In order to allow for the working capital channel to have an effect, we calibrated the weight of wage-bill loans (ν^F) to 0.2.

In addition to the calibration of the parameters that govern the model's steady-state solution, we also set $\chi_{W,g_z} = \gamma_{IM^C} = \gamma_{IM^I} = \gamma_{IM^G} = \gamma_{IM^X} = 0$. In other words, we assume that wages are indexed to the steady-state productivity growth rate rather than to the actual rate, and that there are no adjustment costs in final goods production. Finally, by setting the cost of variation ($\gamma_{u,2}$) to $10,000$ we do not allow capital utilization to vary.

We calibrated the (non-structural) long-run annual secular growth rate in per capita hours worked to 0.7%, which is approximately the sample average. Together with the growth in labor productivity, this amounts to a long-run annual growth rate of 1.7% in per capita GDP, which is approximately the sample average. The annual long-run growth rates of foreign GDP ($g_{\Delta Y^*}$), world trade ($g_{\Delta WT^*}$), per capita private consumption ($g_{\Delta C}$), per capita fixed capital investment ($g_{\Delta I}$), per capita exports ($g_{\Delta X}$) and per capita imports

value only. Christiano et al. (2007) also introduce imports in the production of exports.

[25] In Israel, housing services and fresh fruits and vegetables, comprising 22% of the consumption basket, are not subject to VAT.

Table 1: Calibrated Parameters

Parameter		Value	Parameter		Value
Discount factor	β	0.995	Wage markup	φ^W	1.3
Inverse of labor EoS	ζ	2.0	Dom. price markup	φ^H	1.3
Capital share in prod.	α	0.33	Imp. price markup	φ^*	1.3
LR productivity growth	g_z	1.0025	Exp. price markup	φ^X	1.1
Depreciation rate	δ	0.02	Home bias - C	ν^C	0.65
EoS in consumption	μ^C	1.1	Home bias - I	ν^I	0.60
EoS in investment	μ^I	1.1	Home bias - G	ν^G	0.95
EoS in government	μ^G	0.2	Home bias - X	ν^X	0.68
EoS in exports	μ^X	1.1	Gov. to GDP	s_G	0.26
Foreign EoS	μ^*	1.5	Consumption tax	τ^C	0.16
X's competitors price	p_X^C	1.0	Capital tax	τ^K	0.50
Relative technology	$\widetilde{z}$	1.0	Labor income tax	τ^N	0.28
X's weight in IM^*	ν^*	0.005	Payroll tax - h	τ^{W_h}	0.09
Working capital weight	ν^F	0.2	Payroll tax - f	τ^{W_f}	0.07
Foreign transfers to GDP	s_{FTR}	0.03	Gov. transfers to GDP	s_{TR}	0.15
LR inflation rate	$\overline{\Pi}$	1.005	Δ Inventories in GDP	Δinv	0.01
Share of taxed goods	w_{τ^C}	0.78			

($g_{\Delta IM}$) were calibrated to 2.2%, 5.3%, 1.7%, 1.7%, 4.9% and 2.9%, respectively. These rates are based on the sample averages and an assessment that the sample's excess growth rates (on average) in investment and consumption are transitory phenomena. This calibration implies an annual long-run growth rate of (-1%) in per capita government consumption ($g_{\Delta G}$) during the sample period.

4.4.2 Structural shocks

The estimation of the model, which is based on 24 series of observable variables, involved the following 25 structural shocks, in addition to the 14 observation equation shocks described in appendix B. The structural shocks are assumed to follow first-order auto-regressive processes,[26] apart from the interest rate (Taylor rule) shock which is assumed to be *i.i.d.* Appearing in parentheses are the shock's symbol, its auto-regressive coefficient and its *i.i.d.* innovation.

- Transitory technology (ε_t, ρ, η_t)
- Permanent labor-productivity ($g_{z,t}$, ρ_{g_z}, $\eta_t^{g_z}$)
- Symmetric premium (ε_t^{RP}, ρ_{RP}, η_t^{RP})
- External premium ($\varepsilon_t^{RP^*}$, ρ_{RP^*}, $\eta_t^{RP^*}$)
- Domestic premium (ε_t^{DRP}, ρ_{DRP}, η_t^{DRP})
- Consumption demand (ε_t^{C}, ρ_{C}, η_t^{C})
- Investment technology (ε_t^{I}, ρ_{I}, η_t^{I})
- Inventory investment (Δinv_t, $\rho_{\Delta inv}$, $\eta_t^{\Delta inv}$)
- Government consumption (ε_t^{G}, ρ_{G}, η_t^{G})
- Time-varying export share (ν_t^*, ρ_{ν^*}, $\eta_t^{\nu^*}$)
- Time-varying home bias (ν_t, ρ_{ν}, η_t^{ν})
- Domestic price markup (φ_t^{H}, ρ_{H}, η_t^{H})
- Import price markup (φ_t^{*}, ρ^{*}, η_t^{*})
- Wage markup (φ_t^{W}, ρ_{W}, η_t^{W})
- Export price markup (φ_t^{X}, ρ_{X}, η_t^{X})
- Foreign transfers ($s_{FTR,t}$, ρ_{FTR}, η_t^{FTR})
- Interest rate (η_t^{R})
- Inflation target ($\eta_t^{\overline{\Pi}}$)
- VAT rate ($\eta_t^{\tau^C}$)
- World demand ($\varepsilon_t^{Y^*}$, ρ_{Y^*}, $\eta_t^{Y^*}$)
- World cost push ($\varepsilon_t^{\Pi^*}$, ρ_{Π^*}, $\eta_t^{\Pi^*}$)
- World interest rate ($\varepsilon_t^{R^*}$, ρ_{R^*}, $\eta_t^{R^*}$)
- World trade ($\varepsilon_t^{WT^*}$, ρ_{WT^*}, $\eta_t^{WT^*}$)
- Oil price (ε_t^{OIL}, ρ_{OIL}, η_t^{OIL})
- World LR rate ($\eta_t^{*,fwd}$)

[26] See footnote 4.

The labor supply shock (ε_t^N) is not used since it is observationally equivalent to the wage markup shock (φ_t^W). In order to reduce the number of estimated free parameters (which are under-identified given the data), we assume that the home bias variables in all the final good sectors (ν_t^C, ν_t^G, ν_t^I, ν_t^X) share the same innovation (η_t^ν) and auto-regressive coefficient (ρ_ν), i.e. there is one general home bias process common to all final goods sectors. Hence, η_t^ν may be thought of as a general negative import demand shock. Consequently, we do not use the alternative import demand shock (ε_t^{IM}) since it directly affects the prices of final goods through the import-intensity adjustment cost term in the price equations, an effect that does not seem to have an appealing interpretation in reality.[27] We assume no variation in the relative foreign price of export competitors ($p_{X,t}^{c,*} = P_{X,t}^{c,*}/P_t^*$) and do not use the export demand shock that works through the adjustment costs for the share of exports in world trade (ε_t^X) since they are observationally equivalent to the time varying export share (ν_t^*).[28,29] We assume that the social security tax rates ($\tau_t^{W_h}$ and τ^{W_f}), the capital income tax rate (τ_t^K) and the dividend income tax rate (τ_t^D) are constant, owing to data limitations. Estimations using the direct income tax rate (τ_t^N) yielded unsatisfactory results, probably due to its significant downward trend during the sample period.

4.4.3 Prior distributions

The prior distributions chosen for the estimated parameters are summarized in Table 2 (with the estimation results).[30] Following the common practice in Bayesian estimation of DSGE models, the prior shape (functional form) is chosen according to the feasible support for the parameter. Thus, for parameters that are bounded between 0 and 1 (such

[27] In contrast to the adjustment costs, the effect of ν_t vanishes in the log-linearized form of the final goods price equations (such as 45), provided that the prices of domestic and imported intermediate goods are the same in the steady state.

[28] In the linearized version of the model.

[29] Competitors' prices are not used since the data does not enable us to distinguish between the prices of export competitors and foreign prices in general.

[30] Priors and estimation results for the parameters of the observation equations, as well as the shocks' standard deviations, are reported in Table 3 and the parameters of the world model are reported in Table 4.

as Calvo probabilities, auto-regressive coefficients and various weights), we use the beta distribution; for parameters that must be positive (such as adjustment cost parameters), we use the gamma distribution; for unbounded parameters, we use the normal distribution; and for the standard errors of the shocks, we use the inverse gamma distribution.

Prior means were chosen according to estimation results for similar models in other countries and on the basis of pre-estimation calibration exercises. In these calibration exercises, we parameterized the model based on our evaluation of the resultant model properties, mainly using various impulse response functions, model-based moments and historical shocks decomposition. The main reference models were NAWM for Europe,[31] RAMSES for Europe[32] and Sweden,[33] NEMO for Norway[34] and TOTEM for Canada.[35] All of these models are New Keynesian DSGE models for small open economies, with price and wage rigidities, incomplete exchange rate passthrough, endogenous investment dynamics and the nominal interest rate serving as the instrument of monetary policy.

The prior mean for the habit formation parameter (κ) was set to 0.7, which is in the mid-range of the reference models' estimates (0.57-0.88).

All prior means for the indexation parameters (χ) were set to 0.4, which is between the values typically found by NAWM (0.5) and RAMSES (0.2). It is also in the vicinity of the value estimated by Binyamini (2007) for price indexation in Israel.

The prior means for the price and wage Calvo probability parameters (ξ) were set to 0.6, which corresponds to an optimized price duration of 2.5 quarters. This is somewhat shorter than what is usually reported in macro-based studies (i.e. 3-4 quarters), which is appropriate in view of the the high volatility of inflation in Israel relative to other Western countries and is consistent with estimations carried out by Ribon (2004) and Binyamini (2007).

[31] Christoffel et al. (2008).
[32] Adolfson et al. (2007).
[33] Adolfson et al. (2008).
[34] Brubakk et al. (2006).
[35] Murchison and Rennison (2006).

The prior means for the adjustment cost parameters (γ_I, ω_{Γ_I}, γ^*) were based on our pre-estimation calibration exercises.

The prior means for the parameters of the external financial intermediation premium were set as follows: the coefficient on net foreign assets (γ_B) was set to the NAWM result (0.1) while the coefficient on the exchange rate (γ_S) was set to 0.45, which is somewhat lower than that reported by RAMSES for Sweden (0.6) and was based on the calibration exercise. The prior for the oil import share was set to 0.15, which is approximately the weight of fuels within Israel's total imports of goods..

The priors for the interest rate rule were set according to standard values appearing in the literature (which are in the neighborhood of those found by other studies of the Israeli economy): 0.7 for the smoothing parameter (ϕ_R), 2.5 for the response to inflation (ϕ_Π) and 0.2 for the response to the output gap (ϕ_y). The response parameters are somewhat higher than Taylor's original values (1.5, 0.125);[36] however, note that since we have introduced interest rate smoothing, the overall short-run elasticity is in fact lower than the response parameters. The prior of 0.2 for the interest rate response to a nominal depreciation ($\phi_{\Delta S}$) is not the standard value in the literature and is based in part on previous estimations of interest rate rules for Israel (see, for example, Argov and Elkayam (2010)).

In general, prior means for the various auto-regressive coefficients (ρ) were set to 0.7 (NAWM uses 0.75, and RAMSES uses 0.85), except for the persistence coefficients of the four markup shocks, which were set to 0.3. This reflects our a-priori expectation that markup shocks, which are the residuals of the inflation equations, are unpredictable.

The priors' standard deviations reflect our confidence in the prior means. The larger the standard deviation, the more we allow the posterior distribution to be affected by the likelihood shape. In general, we tried to allow wide priors (that is, high standard deviations). However, we set the standard deviations small enough to induce a single

[36] Taylor's famous 0.5 value for the response to the output gap relates to the annualized interest rate. Since the interest rate in our equation is expressed in quarterly terms, the equivalent parameter is 0.125.

mode, i.e. to enhance the curvature of the posterior distribution.

4.5 Estimation results

The results of the Bayesian estimation are reported in Tables 2, 3 and 4. Table 2 presents the structural parameters and the auto-regressive coefficients of the shocks; Table 3 reports the results for the parameters of the observation equations (see section 4.2 and appendix B) and the standard deviations of the shocks; and Table 4 presents the results for the parameters of the foreign economy model (see section 3.6). In each Table, the middle panel specifies the prior's shape, mean and standard deviation, while the right panel presents statistics for the posterior distribution. The mode was retrieved by standard optimization algorithms and the standard deviation is approximated by the inverse of the Hessian matrix. The mean, as well as the 5th and 95th percentiles of the posterior distribution were calculated by generating 4 chains of 700,000 draws (half of which were burnt out) from the posterior distribution using the Metropolis-Hasting algorithm. The scaling factor of the algorithm was calibrated so as to obtain an acceptance rate of approximately 0.3. The convergence of the chains was monitored using the Brooks and Gelman (1998) algorithm.

Figure 3 depicts the prior and posterior distributions. Cases in which the posterior distribution is similar in location and dispersion to the prior are an indication that the data is poorly informative with regards to the respective parameter (i.e. the likelihood function is fairly flat with respect to this parameter in the region searched). It is evident from figure 3 that most, though not all, parameters are identified by the data. The poorly-identified parameters, for which the posterior essentially replicates the prior, include mainly persistence parameters (ρ_{g_z}, ρ_G, ρ_{ν^*}, ρ^*) and indexation parameters (the various χ's).

The data points to a relatively low degree of price stickiness, i.e. low ξ's in comparison to the estimates typically found for other countries. This is particularly the case in the import sector, where the estimated parameter corresponds to an average optimized price duration of 1.8 quarters, thus indicating a rapid passthrough from the exchange rate and

Table 2: Prior and Posterior Distributions of the Main Structural Parameters

Parameter		Prior distribution type	mean	std	Posterior distribution mode	std	mean	5%	95%
Habit formation	κ	beta	0.70	0.15	0.616	0.064	0.706	0.568	0.861
Indexation parameters									
Employment	χ_E	beta	0.40	0.10	0.494	0.102	0.485	0.316	0.648
Dom. prices	χ_H	beta	0.40	0.10	0.365	0.097	0.355	0.201	0.504
Import prices	χ_{IM}	beta	0.40	0.10	0.300	0.089	0.322	0.179	0.462
Wages	χ_W	beta	0.40	0.10	0.377	0.100	0.377	0.217	0.531
Exports	χ_X	beta	0.40	0.10	0.281	0.085	0.294	0.158	0.429
Calvo parameters									
Employment	ξ_E	beta	0.60	0.10	0.614	0.040	0.646	0.552	0.743
Dom. prices	ξ_H	beta	0.60	0.10	0.606	0.053	0.648	0.552	0.746
Import prices	ξ_{IM}	beta	0.60	0.10	0.428	0.048	0.443	0.361	0.526
Wages	ξ_W	beta	0.60	0.10	0.456	0.057	0.543	0.421	0.664
Exports	ξ_X	beta	0.60	0.10	0.588	0.047	0.596	0.510	0.679
Adj. cost inv.	γ_I	gamma	2.00	1.00	2.816	0.709	3.305	1.919	4.680
Adj. cost inv. lag	ω_{Γ_I}	beta	0.50	0.15	0.554	0.082	0.536	0.394	0.681
Adj. cost export	γ^*	gamma	1.20	0.50	0.295	0.125	0.645	0.154	1.176
FX premium - B*	γ_B	gamma	0.01	0.01	0.012	0.003	0.012	0.006	0.017
FX premium - S	γ_S	beta	0.45	0.20	0.325	0.077	0.358	0.229	0.487
Oil import share	ω^*	beta	0.15	0.05	0.118	0.024	0.133	0.086	0.177
Monetary policy									
Smoothing	ϕ_R	beta	0.70	0.10	0.814	0.035	0.833	0.780	0.887
Resp. to inflation	ϕ_Π	gamma	2.50	0.50	2.538	0.400	2.656	1.942	3.361
Resp. to output	ϕ_y	gamma	0.20	0.10	0.204	0.057	0.205	0.100	0.311
Resp. to depreciation	$\phi_{\Delta S}$	gamma	0.20	0.10	0.090	0.043	0.124	0.037	0.206
Autoregressive coeff.									
Transitory techn.	ρ	beta	0.70	0.15	0.920	0.039	0.859	0.760	0.959
Permanent techn.	ρ_{g_z}	beta	0.70	0.15	0.693	0.161	0.668	0.454	0.900
Symmetric prem.	ρ_{RP}	beta	0.70	0.15	0.767	0.065	0.737	0.575	0.877
External prem.	ρ_{RP^*}	beta	0.70	0.15	0.582	0.105	0.550	0.375	0.727
Consumption	ρ_C	beta	0.70	0.15	0.782	0.241	0.584	0.275	0.938
Inv. techn.	ρ_I	beta	0.70	0.15	0.906	0.035	0.732	0.482	0.944
Inventory inv.	$\rho_{\Delta INV}$	beta	0.70	0.15	0.708	0.109	0.678	0.513	0.852
Government	ρ_G	beta	0.70	0.15	0.679	0.218	0.672	0.416	0.935
Export share	ρ_{ν^*}	beta	0.70	0.15	0.839	0.094	0.664	0.377	0.921
Home bias	ρ_ν	beta	0.70	0.15	0.802	0.091	0.770	0.627	0.915
Domestic markup	ρ_{φ^H}	beta	0.30	0.15	0.196	0.131	0.241	0.039	0.429
Import markup	ρ_{φ^*}	beta	0.30	0.15	0.203	0.135	0.258	0.048	0.461
Wage markup	ρ_{φ^W}	beta	0.30	0.15	0.109	0.079	0.187	0.025	0.343
Export markup	ρ_{φ^X}	beta	0.30	0.15	0.102	0.078	0.142	0.017	0.261
Foreign transfers	ρ_{FTR}	beta	0.70	0.15	0.431	0.183	0.441	0.201	0.681

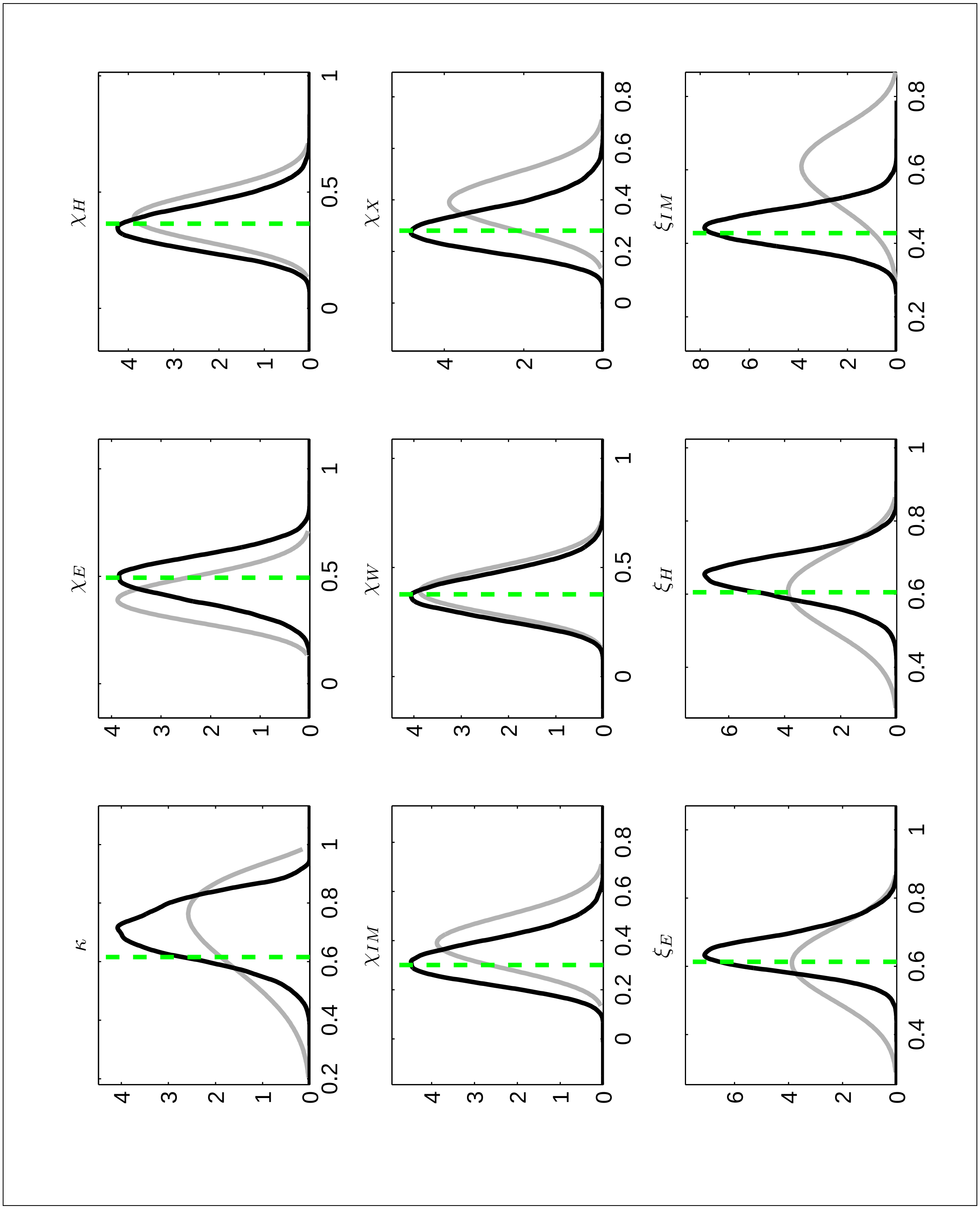
Table 3: Prior and Posterior Distributions of Observation Parameters and Shock Standard Deviations

Parameter		Prior distribution type	mean	std	Posterior distribution mode	std	mean	5%	95%
Observation parameters									
Obs. error output def.	$\rho_{OB,\Delta Y}$	normal	0.00	0.25	-0.185	0.136	-0.189	-0.421	0.032
Employment	$\rho_{OB,E}$	beta	0.70	0.15	0.907	0.044	0.837	0.718	0.959
Constant	tp	normal	0.01	0.01	0.012	0.004	0.011	0.004	0.018
Obs. error	$\rho_{OB,fwd}$	beta	0.70	0.15	0.740	0.169	0.716	0.510	0.938
Constant	tp^*	normal	0.02	0.01	0.018	0.004	0.018	0.011	0.025
Obs. error	ρ_{OB,fwd^*}	beta	0.70	0.15	0.832	0.104	0.791	0.631	0.958
Hours	ρ^N_{EX}	beta	0.70	0.15	0.933	0.038	0.792	0.569	0.979
Consumption	ρ^C_{EX}	beta	0.70	0.15	0.546	0.179	0.600	0.365	0.842
Investment	ρ^I_{EX}	beta	0.70	0.15	0.735	0.189	0.737	0.515	0.948
Export	ρ^X_{EX}	beta	0.70	0.15	0.602	0.169	0.580	0.332	0.824
Import	ρ^{IM}_{EX}	beta	0.70	0.15	0.675	0.160	0.615	0.377	0.855
Foreign GDP	$\rho^{Y^*}_{EX}$	beta	0.70	0.15	0.655	0.143	0.645	0.432	0.866
World trade	$\rho^{WT^*}_{EX}$	beta	0.70	0.15	0.849	0.077	0.800	0.654	0.950
Oil price	$\rho^{P^*_{OIL}}_{EX}$	beta	0.50	0.07	0.566	0.067	0.552	0.443	0.660
Shocks' standard deviations									
Obs. error output def.	S.D.($\eta^{\Delta P^M_Y}$)	inv. gamma	0.01	Inf	0.007	0.001	0.007	0.006	0.009
Employment	S.D.($\eta^{OB,E}$)	inv. gamma	0.01	Inf	0.003	0.000	0.003	0.002	0.003
Dom. term prem.	S.D.($\eta^{fwd,OB}$)	inv. gamma	0.01	Inf	0.002	0.001	0.002	0.002	0.003
Foreign term prem.	S.D.($\eta^{*,fwd,OB}$)	inv. gamma	0.01	Inf	0.002	0.000	0.002	0.001	0.002
Hours worked	S.D.(η^N_{EX})	inv. gamma	0.00	0.00	0.001	0.000	0.001	0.000	0.001
Consumption	S.D.(η^C_{EX})	inv. gamma	0.01	0.00	0.003	0.000	0.003	0.002	0.004
Investment	S.D.(η^I_{EX})	inv. gamma	0.01	0.00	0.004	0.001	0.004	0.003	0.006
Export	S.D.(η^X_{EX})	inv. gamma	0.01	0.00	0.003	0.001	0.003	0.002	0.005
Import	S.D.(η^{IM}_{EX})	inv. gamma	0.01	0.00	0.003	0.001	0.003	0.002	0.004
Wages	S.D.(η^W_{EX})	inv. gamma	0.01	0.00	0.010	0.002	0.011	0.007	0.016
Foreign GDP	S.D.($\eta^{Y^*}_{EX}$)	inv. gamma	0.01	0.00	0.003	0.000	0.003	0.002	0.003
World trade	S.D.($\eta^{WT^*}_{EX}$)	inv. gamma	0.01	0.00	0.004	0.001	0.004	0.003	0.006
Oil price	S.D.($\eta^{P^*_{OIL}}_{EX}$)	inv. gamma	0.03	0.01	0.032	0.007	0.038	0.023	0.052
Exchange rate	S.D.(η^S_{EX})	inv. gamma	0.01	0.01	0.013	0.002	0.014	0.010	0.018
Transitory techn.	S.D.(η)	inv. gamma	0.03	Inf	0.011	0.001	0.012	0.010	0.013
Permanent techn.	S.D.(η^{g_z})	inv. gamma	0.01	Inf	0.002	0.000	0.002	0.001	0.002
Symmetric prem.	S.D.(η^{RP})	inv. gamma	0.03	Inf	0.010	0.003	0.013	0.006	0.022
External prem.	S.D.(η^{RP^*})	inv. gamma	0.03	Inf	0.011	0.002	0.011	0.008	0.014
Dom. prem.	S.D.(η^{DRP})	inv. gamma	0.00	Inf	0.000	0.000	0.001	0.000	0.001
Consumption	S.D.(η^C)	inv. gamma	0.03	Inf	0.012	0.006	0.035	0.007	0.067
Inv. techn.	S.D.(η^I)	inv. gamma	0.05	Inf	0.042	0.009	0.052	0.020	0.081
Inventory inv.	S.D.($\eta^{\Delta INV}$)	inv. gamma	0.01	Inf	0.013	0.001	0.013	0.011	0.015
Government	S.D.(η^G)	inv. gamma	0.01	Inf	0.006	0.001	0.006	0.004	0.008
Export share	S.D.(η^{ν^*})	inv. gamma	0.05	Inf	0.042	0.007	0.058	0.035	0.082
Home bias	S.D.(η^{ν})	inv. gamma	0.01	Inf	0.008	0.001	0.008	0.007	0.010
Domestic markup	S.D.(η^{φ^H})	inv. gamma	0.05	Inf	0.028	0.009	0.038	0.016	0.059
Import markup	S.D.(η^{φ^*})	inv. gamma	0.03	Inf	0.026	0.007	0.029	0.015	0.043
Wage markup	S.D.(η^{φ^W})	inv. gamma	0.50	Inf	0.238	0.068	0.335	0.141	0.519
Export markup	S.D.(η^{φ^X})	inv. gamma	0.10	Inf	0.066	0.017	0.076	0.042	0.107
Foreign transfers	S.D.(η^{FTR})	inv. gamma	0.01	Inf	0.016	0.002	0.016	0.013	0.019
Interest rate	S.D.(η^R)	inv. gamma	0.01	Inf	0.002	0.000	0.002	0.002	0.003
Inf. target	S.D.($\eta^{\overline{\Pi}}$)	inv. gamma	0.00	Inf	0.001	0.000	0.001	0.001	0.001
Consumption tax	S.D.(η^{τ^C})	inv. gamma	0.00	Inf	0.002	0.000	0.002	0.002	0.003

Table 4: Prior and Posterior Distributions of the World Model's Parameters

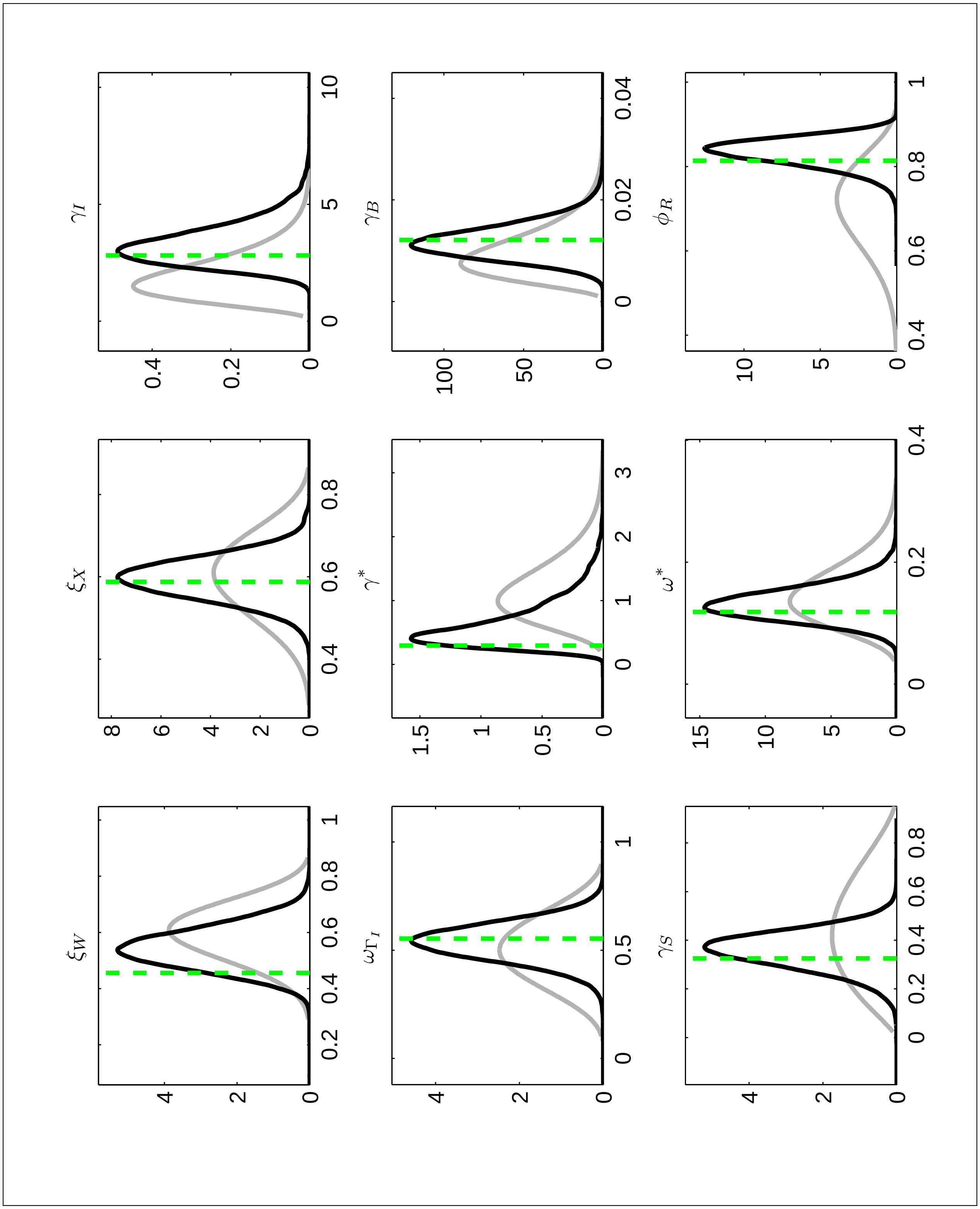
Parameter		Prior distribution type	Prior mean	Prior std	Posterior mode	Posterior std	Posterior mean	5%	95%
Output equation									
Expectations	$c_{y^*,+}$	beta	0.50	0.10	0.227	0.048	0.225	0.146	0.302
Real rate	$c_{y^*,r}$	gamma	0.20	0.05	0.156	0.030	0.157	0.107	0.205
AR in shock	ρ_{Y^*}	beta	0.70	0.07	0.643	0.063	0.636	0.535	0.738
Inflation equation									
Expectations	$c_{\pi^*,+}$	beta	0.70	0.15	0.966	0.026	0.953	0.915	0.993
Output	$c_{\pi^*,y}$	gamma	0.10	0.03	0.071	0.017	0.076	0.046	0.105
Oil - level	$c_{\pi^*,OIL}$	gamma	0.05	0.03	0.025	0.007	0.028	0.015	0.041
Oil - change	$c_{\pi^*,\Delta OIL}$	gamma	0.05	0.03	0.016	0.005	0.017	0.008	0.026
AR in shock	ρ_{Π^*}	beta	0.70	0.15	0.247	0.084	0.261	0.125	0.391
Monetary policy equation									
Smoothing	$c_{r^*,-}$	beta	0.75	0.10	0.832	0.033	0.831	0.778	0.885
Resp. to inflation	$c_{r^*,\pi}$	gamma	2.50	0.50	2.087	0.429	2.211	1.487	2.908
Resp. to output	$c_{r^*,y}$	gamma	0.50	0.05	0.516	0.051	0.523	0.437	0.606
AR in shock	ρ_{R^*}	beta	0.25	0.05	0.302	0.053	0.301	0.215	0.385
World trade equation									
Output	$c_{wt,y}$	normal	2.50	0.50	2.336	0.371	2.331	1.719	2.961
Lagged output	$c_{wt,y-}$	normal	0.00	1.00	1.509	0.746	1.380	0.165	2.626
Lagged world trade	$c_{wt,-}$	normal	0.00	0.50	0.176	0.149	0.215	-0.029	0.468
Oil price inflation									
Lag	$c_{oil,-}$	beta	0.70	0.15	0.667	0.078	0.635	0.505	0.771
Lagged change	$c_{oil,\Delta}$	normal	0.00	0.50	-0.415	0.096	-0.411	-0.576	-0.250
Shocks' standard deviations									
Demand	S.D.(η^{Y^*})	inv. gamma	0.01	Inf	0.002	0.000	0.002	0.002	0.003
Supply	S.D.(η^{Π^*})	inv. gamma	0.01	Inf	0.007	0.001	0.007	0.006	0.009
Interest rate	S.D.(η^{R^*})	inv. gamma	0.00	Inf	0.002	0.000	0.002	0.002	0.003
World trade	S.D.(η^{WT^*})	inv. gamma	0.03	Inf	0.007	0.001	0.007	0.005	0.009
Oil price	S.D.(η^{OIL})	inv. gamma	0.10	Inf	0.124	0.012	0.124	0.101	0.146
LR rate	S.D.($\eta^{*,fwd}$)	inv. gamma	0.00	Inf	0.001	0.000	0.001	0.000	0.001

Figure 3: Prior and Posterior Distributions of the Structural Parameters



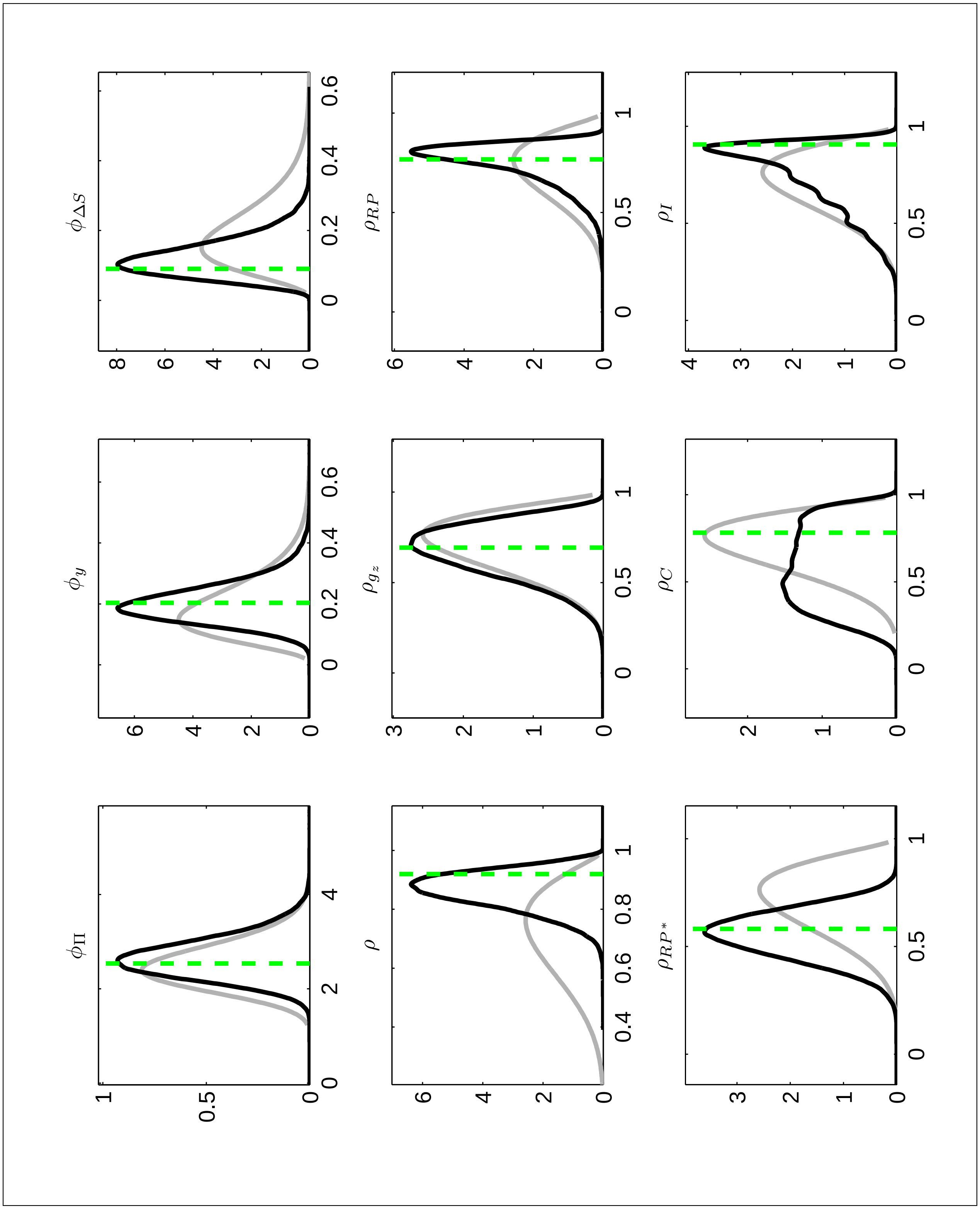
Note: Dashed line indicates the conditional posterior mode. Black line indicates the posterior distribution. Gray line indicates the prior distribution.

Figure 3: Prior and Posterior Distributions of the Structural Parameters (cont.)



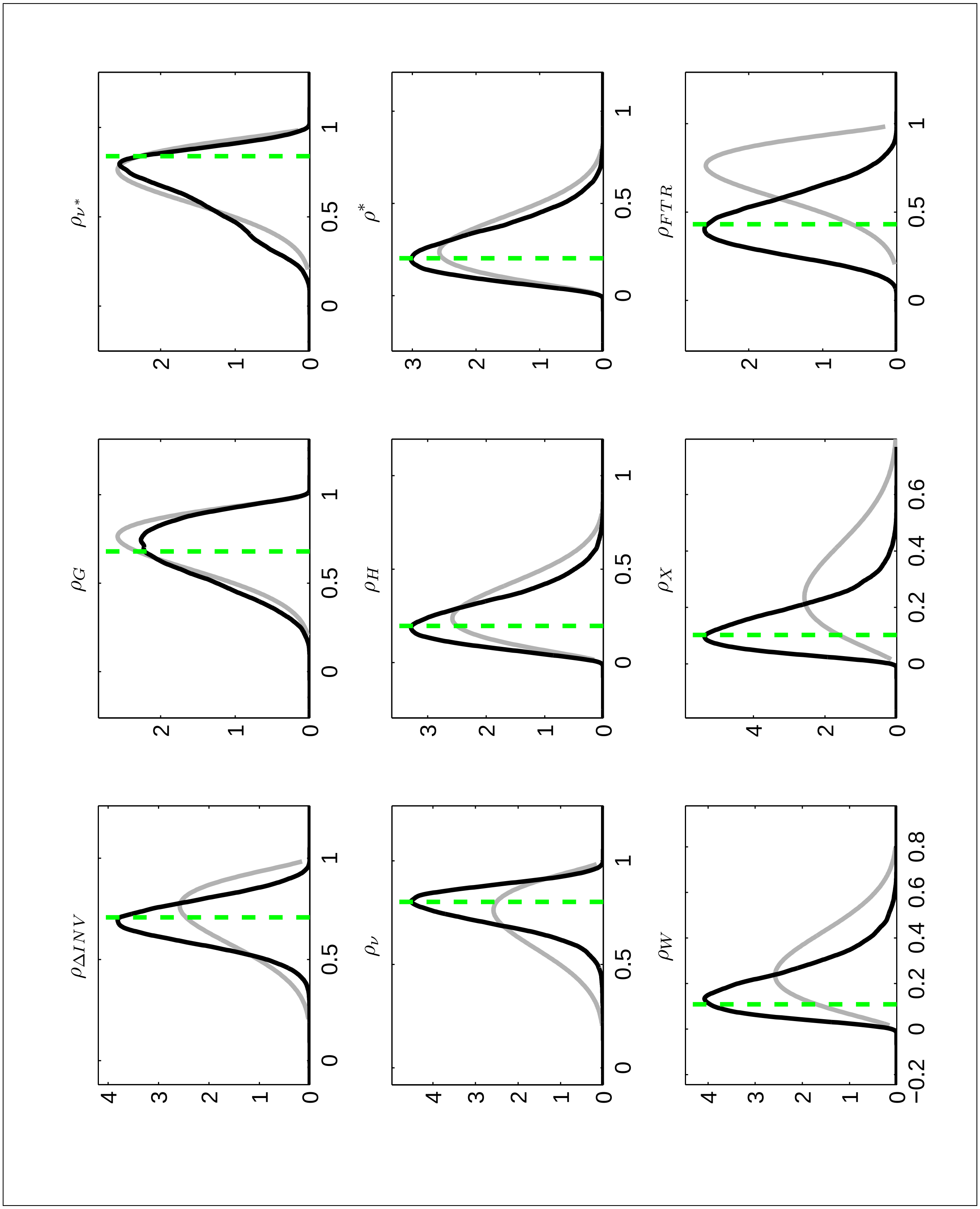
Note: Dashed line indicates the conditional posterior mode. Black line indicates the posterior distribution. Gray line indicates the prior distribution.

Figure 3: Prior and Posterior Distributions of the Structural Parameters (cont.)



Note: Dashed line indicates the conditional posterior mode. Black line indicates the posterior distribution. Gray line indicates the prior distribution.

Figure 3: Prior and Posterior Distributions of the Structural Parameters (cont.)



Note: Dashed line indicates the conditional posterior mode. Black line indicates the posterior distribution. Gray line indicates the prior distribution.

foreign prices to domestic import prices. Notice that the estimated stickiness in wages (corresponding to an optimized wage duration of 2.2 quarters) is lower than our prior (2.5 quarters) and lower than the estimated rigidity in domestic prices (2.8 quarters). However, this result may partly reflect the volatility of the aggregate wage data rather than the flexibility of nominal wages.

The data appears to be consistent with our prior of 0.7 for habit persistence, which is a typical value used in similar models. On the other hand, the data indicates higher adjustment costs for investment (γ_I) than our prior; nonetheless, the posterior mean (3.3) is still lower than those obtained by the NAWM (5.2) and RAMSES (8.7). The data does not support the existence of an adjustment cost for the composition of world imports (γ^*); its posterior mean turned out to be 0.65, half of the prior mean. Although the data supports the modification of the UIP condition, introduced through the parameter γ_S (see equation (4) and the discussion there), its posterior mean (0.36) is somewhat lower than our prior (0.45) and the posterior median (0.61) found for Sweden by Adolfson et al. (2008).

Most of the parameters of the monetary policy rule are well-identified by the data. The data provides firm support for interest rate smoothing and yielded a posterior mean of 0.83 for ϕ_R, which is a typical value for extended Taylor-type rules. Our prior of 0.2 for the output gap reaction coefficient (ϕ_y) receives some support from the data (as reflected by a posterior distribution that is somewhat narrower than the prior), while the posterior mean of the exchange rate reaction parameter ($\phi_{\Delta S}$) is somewhat lower than our prior (0.12 as compared to 0.2). Unfortunately, the data is ambiguous with regard to the inflation reaction parameter (ϕ_Π), with the prior distribution (mean of 2.5) approximately retrieved by the posterior. Nevertheless, our estimate largely conforms with the estimates obtained for other countries, as well as previous estimates obtained for Israel.

Regarding the estimated persistence of shocks, technology shocks appear to be relatively persistent, with the highest modes (approximately 0.9) found for the auto-regressive coefficients of the transitory technology shock (ρ) and of the investment specific technology shock (ρ_I) . This may reflect a relatively important role for supply factors in the explanation of Israel's business cycle during the sample period. The persistence parameters with the lowest posterior modes (0.1-0.2) are those of the various markup shocks. Note that while this is primarily a result of our lower priors for these parameters, it also reflects information inherent in the data (since the posteriors are lower than the priors). Our estimate of the persistence of the external risk premium shock (ρ_{RP*}) (posterior mean of 0.55, given a prior of 0.7) is in line with Adolfson et al. (2008), who found that introducing the modification to the UIP results in a lower persistence for this shock.[37] Note that lower persistence corresponds to less predictable deviations from the UIP condition.

4.6 Sensitivity analysis

In this section, we examine the sensitivity of the estimation results to variations in the interest rate rule (62). The analysis is motivated by the fact that the interest rate rule is the only ad hoc behavioral equation in the model, whereas the other behavioral equations are typically based on micro-foundations, i.e. optimization by economic agents. The analysis examines the sensitivity of the parameter estimates to different assumptions regarding the structure of the policy rule, as well as the overall fit of the model (as reflected by the marginal likelihood). Since Metropolis-Hasting draws are highly time-consuming, we focus on the posterior mode and estimate the model under five alternative specifications for the interest rate rule.

In alternatives 1 and 2, we allow the central bank to smooth interest rate changes by adding the term ΔR_{t-1} with coefficient $\phi_{\Delta R}$. We set the prior distribution of $\phi_{\Delta R}$ to be Gamma-shaped with mean of 1.0. This is motivated by the literature on optimal policy under commitment, in which the policy maker is risk averse with respect to interest rate

[37] Adolfson et al. (2008) obtain a posterior median of 0.68 (given a prior mean of 0.85) for the model with modified UIP, compared to a much higher median (0.93) for a specification without the modification. Christoffel et al. (2008) obtain a posterior mean of 0.88 with a prior of 0.75 (for a model with non-modified UIP).

volatility.[38] The two alternatives differ in the prior's standard deviation, i.e. in the weight assigned to the data while estimating $\phi_{\Delta R}$. Thus, in alternative 1, we set the prior's standard deviation to 0.5, while in alternative 2 we set a tight prior with a standard deviation of 0.05.

In alternatives 3 and 4, we generalize the policy rule by allowing a response to the output growth rate, in addition to (or instead of) the output gap. Thus, the policy rule now includes the lagged growth rate of output $(\hat{y}_{t-1} - \hat{y}_{t-2} + \hat{g}_{z,t-1})$, with coefficient $\phi_{\Delta Y}$. Note that the additional term is located inside the squared brackets of the rule (62), such that the overall coefficient is $(1 - \phi_R)\,\phi_{\Delta Y}$. The prior distribution is Gamma-shaped with mean and standard deviation of 0.1 and 0.05, respectively. In alternative 3, the response to the growth rate replaces that to the output level, i.e. $\phi_y = 0$, while in alternative 4 we allow for both responses.

Finally, in alternative 5, we specify a fully forward-looking rule. Thus, instead of responding to a combination of lagged and expected inflation, the policy rule (65), responds to:

$$\hat{\pi}_t^{CB} = E_t\left[\hat{\pi}_{C,t} + \hat{\pi}_{C,t+1} + \hat{\pi}_{C,t+2} + \hat{\pi}_{C,t+3}\right].$$

Overall, the results, which are summarized in Table 5, do not change significantly when the specification of the policy rule is varied. Nevertheless, the following results are worth noting:

Allowing for the smoothing of interest rate changes (alternatives 1 and 2), has an effect on the marginal likelihood. Thus, tightening the prior of the coefficient on the lagged change in the interest rate around unity (alternative 2) reduces the fit of the model. This is also reflected in the relatively low posterior mode of 0.234 under a wider prior (alternative 1), with the rest of the parameters (particularly those of the policy rule) and the overall fit

[38] According to the literature (Woodford, 2003, Ch. 8.3), the interest rate rule under such assumptions includes the term ΔR_{t-1} with its coefficient being equal to the inverse of the time discount factor, β^{-1}, which is slightly larger than one.

Table 5: Posterior Mode Sensitivity to Different Interest Rate Rules

Parameter		Baseline	1	2	3	4	5
			$\phi_{\Delta R}$	$\phi_{\Delta R}=1$	$\phi_{\Delta Y}$ $\phi_y=0$	$\phi_{\Delta Y}$	fwd looking
Habit formation	κ	0.616	0.620	0.625	0.619	0.611	0.607
Indexation parameters							
Employment	χ_E	0.494	0.490	0.476	0.502	0.494	0.499
Dom. prices	χ_H	0.365	0.368	0.364	0.375	0.364	0.339
Import prices	χ_{IM}	0.300	0.305	0.318	0.310	0.298	0.285
Wages	χ_W	0.377	0.378	0.377	0.379	0.376	0.387
Exports	χ_X	0.281	0.279	0.273	0.284	0.279	0.275
Calvo parameters							
Employment	ξ_E	0.614	0.615	0.621	0.614	0.614	0.618
Dom. prices	ξ_H	0.606	0.602	0.601	0.608	0.609	0.676
Import prices	ξ_{IM}	0.428	0.423	0.396	0.430	0.429	0.473
Wages	ξ_W	0.456	0.459	0.476	0.447	0.459	0.474
Exports	ξ_X	0.588	0.590	0.594	0.589	0.588	0.593
Adj. cost inv.	γ_I	2.816	2.788	2.528	3.056	2.786	2.541
Adj. cost inv. lag	ω_{Γ_I}	0.554	0.554	0.558	0.554	0.556	0.549
Adj. cost export	γ^*	0.295	0.304	0.322	0.316	0.295	0.287
FX premium - B*	γ_B	0.012	0.012	0.012	0.013	0.012	0.011
FX premium - S	γ_S	0.325	0.352	0.412	0.326	0.334	0.326
Oil import share	ω^*	0.118	0.120	0.122	0.122	0.118	0.118
Monetary policy							
Smoothing	ϕ_R	0.814	0.805	0.743	0.858	0.813	0.837
Resp. to inflation	ϕ_Π	2.538	2.273	1.801	2.681	2.515	2.733
Resp. to output	ϕ_y	0.204	0.209	0.296	-	0.194	0.233
Resp. to depreciation	$\phi_{\Delta S}$	0.090	0.112	0.228	0.116	0.095	0.085
Resp. to R change	$\phi_{\Delta R}$	-	0.234	0.924	-	-	-
Resp. to output growth	$\phi_{\Delta Y}$	-	-	-	0.172	0.149	-
Autoregressive coeff.							
Transitory techn.	ρ	0.920	0.913	0.875	0.926	0.917	0.911
Permanent techn.	ρ_{g_z}	0.693	0.684	0.673	0.796	0.697	0.775
Symmetric prem.	ρ_{RP}	0.767	0.768	0.788	0.622	0.766	0.734
External prem.	ρ_{RP^*}	0.582	0.543	0.427	0.609	0.574	0.587
Consumption	ρ_C	0.782	0.778	0.743	0.758	0.780	0.746
Inv. techn.	ρ_I	0.906	0.904	0.899	0.913	0.904	0.914
Inventory inv.	$\rho_{\Delta INV}$	0.708	0.704	0.695	0.689	0.707	0.697
Government	ρ_G	0.679	0.680	0.677	0.677	0.680	0.684
Export share	ρ_{ν^*}	0.839	0.830	0.810	0.828	0.839	0.861
Home bias	ρ_ν	0.802	0.798	0.788	0.763	0.804	0.779
Domestic markup	ρ_H	0.196	0.202	0.204	0.179	0.195	0.191
Import markup	ρ^*	0.203	0.211	0.232	0.207	0.198	0.196
Wage markup	ρ_W	0.109	0.113	0.120	0.098	0.109	0.109
Export markup	ρ_X	0.102	0.100	0.090	0.108	0.102	0.104
Foreign transfers	ρ_{FTR}	0.431	0.420	0.379	0.435	0.432	0.439
Marginal likelihood		4444.6	4444.8	4429.7	4443.9	4447.5	4439.5

of the model remaining similar to the baseline.

Replacing the output gap with output growth (alternative 3) does not improve the likelihood. The resulting coefficient is 0.172, which is close to that obtained using the output gap (with the smoothing parameter increasing slightly, probably because the growth rate is more volatile than the output gap). Some improvement is achieved relative to the baseline when including both the growth rate and the output gap (alternative 4), with the marginal likelihood increasing from 4444.6 to 4447.5. In this case, the coefficient of the output gap is virtually the same as in the baseline (0.2) and in addition there is a response to the output growth rate of 0.15. The rest of the parameters are similar to the baseline results.

Using a forward-looking rule (alternative 5) does not substantially change the estimated parameters of the policy rule, though it reduces the marginal likelihood to some extent. This interesting result is somewhat at odds with the fact that the Bank of Israel has traditionally emphasized the role of inflation expectations, and particularly market-based expectations, in the conduct of monetary policy. Indeed, single-equation estimations of Taylor-type interest rate rules with market-based expectations usually perform quite well.[39] This is consistent with the fact that market-based inflation expectations may differ from model-consistent ones.

In sum, the estimation results are not particularly sensitive to variations in the policy rule. Some further discussion of the alternative policy rule specifications can be found in section 5.

[39] See, for example, Argov and Elkayam (2010), Beenstock and Ilek (2010), Melnick (2005), Leiderman and Bar-Or (2002) and Sussman (2007).

5 Model evaluation

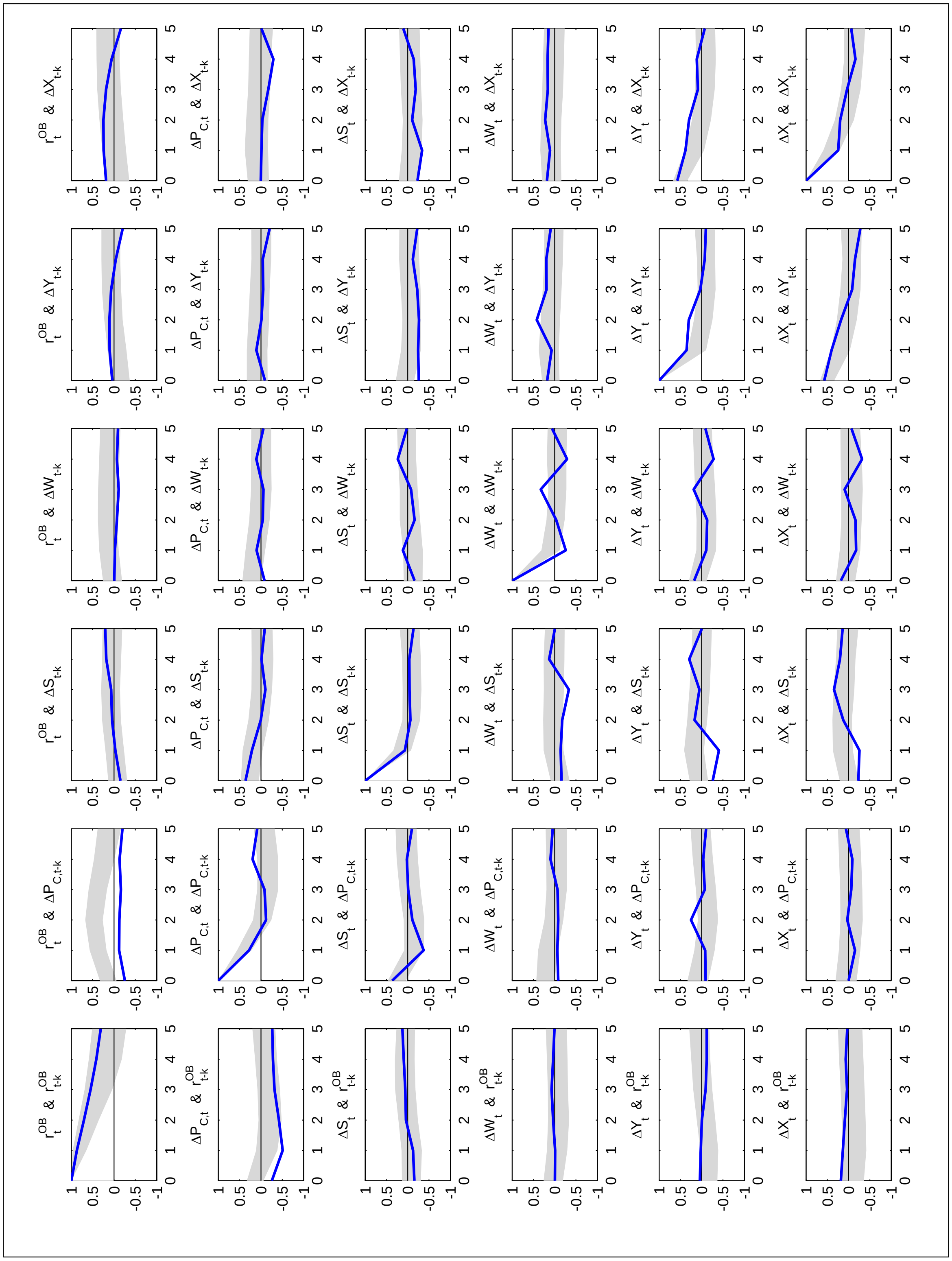
5.1 Moment goodness-of-fit tests

Figure 4 presents the cross- and auto-correlations of the following six key variables, with the corresponding model-based confidence intervals: the nominal interest rate, CPI inflation, the nominal effective depreciation and the growth rates of the nominal hourly wage, per capita output and of per capita exports (r_t^{OB}, $\Delta P_{c,t}$, ΔS_t, ΔW_t, ΔY_t and ΔX_t, respectively). The cross-correlations in the data were calculated for the sample of 72 quarterly observations used in the model's estimation (1992:Q1 to 2009:Q4). In order to compute the model-based confidence intervals, we generated 1,000 simulations of 72 periods each and calculated the cross-correlations for each simulated sample.[40] Thus, for each moment we obtained a distribution of 1,000 estimators. The confidence intervals presented in figure 4 represent the middle 90% of each distribution. It is worth mentioning that for the confidence interval simulation the model included the built-in filtering block (see section 4.2), in order for the simulated moments to be consistent with both the model's description of the business cycle and the imbalanced growth inherent in the data.

The diagrams along the diagonal of figure 4 present the auto-correlations. The model suggests that three of the selected variables have a significant auto-correlation of first order: the nominal interest rate (r_t^{OB}), inflation ($\Delta P_{c,t}$) and the per-capita export growth rate (ΔX_t). Indeed, these theoretical auto-correlations are consistent with the observed ones, which can be seen from the fact that the observed moments, represented by the blue lines, lie within the range of the confidence intervals. At the same time, it appears that the model fails to capture the observed inertia in the per-capita output growth rate (ΔY_t). For the two remaining variables, i.e. the nominal effective depreciation (ΔS_t) and the growth of the nominal hourly-wage (ΔW_t), neither the data nor the model is characterized by significant

[40] All the simulations used the estimated model with parameter values at their posterior means. Thus, the source of uncertainty represented by the confidence intervals is the realization of the shocks and not parameter uncertainty.

Figure 4: Cross-Correlations: Observed vs. Model-Based Confidence Intervals.



Blue lines represent the observed moments. Gray areas represent the model-based confidence intervals (90%). The order of the cross correlation (k) appears in the x-axis.

auto-correlation.

The cross-correlations between the variables appear in the off-diagonal diagrams in figure 4. It can be seen that in most cases, the cross-correlations in the data lie within the model's confidence intervals. A notable exception is the correlation between the interest rate, r_t^{OB}, and lagged inflation, $\Delta P_{c,t-k},\ k \in \{0, ..., 5\}$. While the model suggests significant and positive correlations, this is not observed in the data. Furthermore, computing the observed moments for sub-samples did not generate any positive correlations either.[41] Since the correlation between the interest rate and lagged inflation in the model largely hinges on the central bank's policy rule, and in particular on the reaction of the interest rate to inflation, we examined the cross correlations under alternative rules. Moreover, the central bank policy rule is the only behavioral equation in the model without micro-foundations, and therefore it is a natural candidate for modification in an attempt to improve the moments' fit. Indeed, estimation of the model using some alternative specifications of the policy rule successfully reduced the model-based cross-correlations between r_t^{OB} and $\Delta P_{c,t-k}$. However, alternative specifications that improved the fit of these moments, simultaneously worsened the fit of others. In the final judgment, we chose to remain with the policy rule represented by (62).[42]

In the other direction, i.e. the cross-correlation of inflation $(\Delta P_{c,t})$ with the lagged interest rate (r_{t-k}^{OB}), the negative correlation in the data is captured fairly well, suggesting that the model is consistent with a description of the main transmission mechanisms from monetary policy to inflation.

The model also captures the correlation of inflation with contemporaneous and lagged depreciation, suggesting that the model's passthrough mechanisms (from the exchange rate to prices) may be a reasonable description of reality.

[41] We also examined alternatives for the observed variables: the interest rate in terms of deviations from its long-run forward rate and CPI inflation including housing (for the entire sample). These alternatives did not yield any improvement either.

[42] See section 4.6 for a detailed discussion of the sensitivity of the estimation results to alternative policy rules.

For many of the cross correlations the model fails to generate values that are significantly different from zero. Yet, figure 4 suggests that this is consistent with the real data-generating process.

Table 6: Selected Means and Standard Deviations

Model-Based Intervals vs. the Data

	Mean				Standard Deviation			
Variable	**Median**	**5%**	**95%**	**data**	**Median**	**5%**	**95%**	**data**
ΔC	0.42	0.17	0.72	0.44	1.87	1.55	2.26	1.22
ΔX	1.21	0.78	1.67	1.22	4.37	3.59	5.17	3.62
ΔI	0.43	-0.18	1.05	0.06	3.54	2.96	4.25	3.13
ΔIM	0.71	0.34	1.07	0.56	3.59	3.05	4.21	3.49
ΔY	0.43	0.24	0.61	0.39	1.80	1.51	2.09	1.00
ΔP_c	0.51	0.14	0.87	0.37	0.88	0.73	1.04	0.82
ΔS	0.01	-0.51	0.53	-0.21	3.33	2.84	3.89	3.33
r^{OB}	6.35	-2.15	12.89	5.51	2.57	1.85	3.48	2.49
ΔW	0.75	0.25	1.23	0.61	1.82	1.56	2.12	1.68
ΔN	0.17	0.02	0.32	0.14	2.46	2.10	2.83	1.38
s_{CA}	-0.02	-1.28	1.35	0.40	4.03	2.99	5.56	3.00

Note: Model-based intervals are based on 1,000 simulations of 72 periods each.

In order to complete the moment goodness-of-fit tests, Table 6 compares the means and standard deviations of selected variables to their model-based intervals. As can be seen, the observed means fall well within the model-based intervals for all the variables. This is also the case for most of the standard deviations, and for the few exceptions the observed standard deviations are below their respective model-based intervals. Two such notable exceptions are the standard deviations of output and hours worked growth rates (ΔY and ΔN, respectively), which can partly be explained by the exogenous law of motion specified for the inventory rate of change (3). Thus, in practice, a change in inventory acts as a buffer by smoothing the supply of demand-determined output. However, once we limit its law of motion to be exogenous, inventory becomes another source of volatility, primarily influencing output and production activity.[43] Another noteworthy result in Table 6 is the exceptionally wide interval for the mean of the interest rate, r^{OB}, which is a result of the unit-root process of the shock to the forward real interest rate, ε_t^{DRP} (which is, again, also reflected in the decomposition of Table 7). Both the unit-root process and the exceptionally wide confidence interval for the mean of r^{OB} are consistent with the fact that the interest rate has an observed trend during the sample period.[44]

5.2 Forecast quality

Figure 5 depicts the Root Mean Squared Errors (RMSE) of the unconditional forecasts of selected variables, up to 8 quarters ahead. These include the BoI interest rate (r^{OB}), the levels of the CPI, output, consumption, nominal effective exchange rate, exports, nominal hourly wages and hours worked (cumulative forecast RMSE of ΔP_C, ΔY, ΔC, ΔS, ΔX, ΔW and ΔN, respectively). For most of the variables, the model-based unconditional forecasts seem to be no worse than the following naive alternatives: steady state (SS), Random Walk (RW) and Bayesian VAR (BVAR) of third order.[45,46]

The RMSEs of the model-based interest rate forecast are lower than the alternatives, suggesting that the model is a better predictor for this variable. Notice that the model-based forecasts of the CPI and output fail to beat the SS-based forecasts, unlike the forecast of the nominal interest rate. This is consistent with the view that under a (flexible) inflation-targeting regime, the interest rate is the main variable that deviates from SS while absorbing

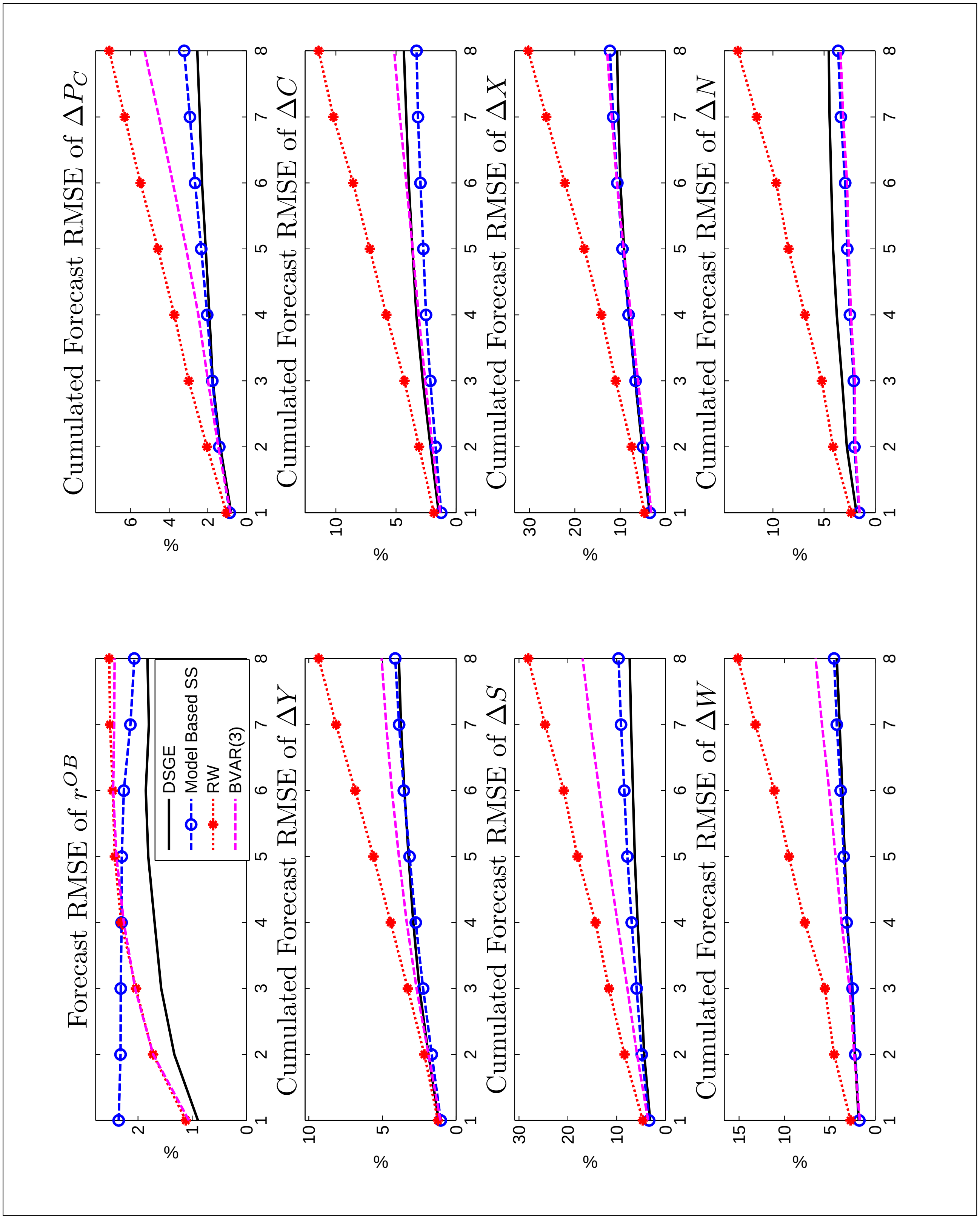
[43] Running the same simulations without drawing from the inventory-change shock shifts the two confidence intervals (for the standard deviations of ΔY and ΔN) downward while hardly affecting the other simulated moments. However, it still leaves the observed standard deviations below the lower boundaries of the new intervals.

[44] Running the same simulations without using the ε_t^{DRP} shock narrows the confidence interval of the interest rate mean to 3.04-6.82, from the much wider interval of (-2.15)-12.89 in Table 6, with a negligible effect on other simulated moments.

[45] For the BVAR, we used Minnesota-like priors, with prior means of 0.5. Using different prior means yielded larger RMSEs for the BVAR-based forecasts.

[46] The BVAR includes all the eight variables presented in figure 5. Adding the rate of change in employment, ΔEM, reduced the RMSE of the BVAR-based forecasts for wages and consumption, ΔW and ΔC (to lower than their model-based counterparts). However, at the same time, it significantly increased the forecast RMSEs for the other variables.

Figure 5: In-Sample Forecast Root Mean Square Errors (RMSE).



Note: Forecast horizon appears on the x-axis.

shocks and thereby stabilizing the rest of the economy.

It appears that the model-based *unconditional* forecast is no worse than alternative naive forecasts. This is an encouraging result since in the actual use of the model as a forecasting tool, we condition the forecast on a great deal of out-of-model information, which is expected to further improve the forecasts.[47] While it is (relatively) easy to incorporate such information in the DSGE model, it is difficult in the BVAR and impossible in the SS and RW alternatives.

We identified two shocks that, if they could have been predicted *ex ante*, would have significantly improved the forecast: ε_t^{RP*}, the shock to the modified UIP condition (7), which has a high standard deviation and strong transmission mechanisms, and ε_t^{RP}, the shock to the risk premium on bonds in the household budget constraint (2), which is highly inertial. Projecting the forecast as if we had *ex ante* information on these two specific shocks would significantly reduce the forecasts' RMSEs.

6 Model properties

6.1 Variance decomposition

Table 7 presents the forecast-error variance decomposition for ten key variables for three horizons: one quarter, four quarters and an infinite horizon. The variables are stationarized, i.e. real variables are divided by z_t. The main shocks contributing to the variance of consumer price inflation (ΔP_C) are the external risk premium shock (33-38%) (through its effect on the exchange rate), markup shocks in the domestic and import sectors (14-21% and 16-23%, respectively) and for the four-quarter and infinite horizons, also oil price shocks (8.7% and 11.5%, respectively). Note that none of these shocks makes a significant contribution to the variance of the major real variables, namely output, the main components of output and hours worked.

[47] On the other hand, Maih (2010) shows that incorporating such future information into (inevitably misspecified) models may worsen forecasts' RMSE.

Table 7: Forecast Error Variance Decompostion

(one-quarter, four-quarter and infinite horizon)

		Interest rate r^{OB}			Inflation ΔP_C			Real exchange rate $\hat{s}$			Real wage $\hat{w}$			Hours worked $\hat{N}$		
		1	4	∞	1	4	∞	1	4	∞	1	4	∞	1	4	∞
Transitory techn.	η	2.6	7.2	2.1	4.1	4.1	4.1	12.1	24.9	19.0	0.3	1.5	10.0	37.9	23.8	20.2
Permanent techn.	η^{g_z}	0.0	0.0	0.2	0.0	0.0	0.3	0.0	0.2	0.5	0.3	0.8	2.6	0.1	0.5	1.4
Symmetric prem.	η^{RP}	0.7	5.5	3.7	0.5	1.2	1.1	4.0	10.1	12.6	5.9	13.6	11.1	7.3	9.3	8.0
External prem.	η^{RP^*}	25.8	42.9	7.0	37.7	35.1	33.0	55.7	24.6	13.0	8.4	7.9	5.0	1.5	3.2	2.7
Dom. prem.	η^{DRP}	0.6	2.3	75.5	0.2	0.3	0.3	0.1	0.1	0.0	0.0	0.0	0.4	0.0	0.1	0.1
Consumption	η^{C}	0.1	0.5	0.3	0.0	0.0	0.1	0.3	0.6	0.4	0.0	0.1	1.1	2.2	3.7	3.4
Inv. techn.	η^{I}	0.1	0.3	0.7	0.0	0.1	2.6	0.0	0.1	8.2	0.0	0.1	6.4	2.1	2.5	4.3
Inventory inv.	$\eta^{\Delta INV}$	0.9	1.4	0.4	0.1	0.1	0.1	0.4	0.4	0.4	0.1	0.0	1.1	14.8	9.6	7.9
Government	η^{G}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.3	0.3
Export share	η^{ν^*}	0.0	0.0	0.3	0.2	0.5	0.8	6.3	12.3	11.8	1.6	5.3	8.2	11.0	13.1	11.2
Home bias	η^{ν}	0.2	0.1	0.2	0.1	0.3	0.5	2.6	5.4	5.6	1.0	2.9	4.4	15.6	11.0	9.2
Domestic markup	η^{H}	1.5	2.6	0.4	20.6	15.5	13.7	4.7	3.1	1.6	8.0	6.4	3.5	1.6	3.6	3.1
Import markup	η^{*}	2.1	2.9	0.4	22.9	18.0	15.8	0.0	0.2	0.1	6.3	2.7	1.4	0.8	0.6	0.5
Wage markup	η^{W}	0.6	2.8	0.6	4.6	5.5	4.8	3.0	6.2	3.8	66.4	49.5	24.2	0.9	6.8	9.4
Export markup	η^{X}	0.0	0.2	0.0	0.0	0.0	0.0	2.7	0.8	0.4	0.1	0.2	0.1	1.0	2.0	1.7
Foreign transfers	η^{FTR}	0.1	0.6	0.4	0.2	0.3	0.4	0.4	0.9	1.4	0.2	0.8	2.7	0.0	0.0	0.2
Interest rate	η^{R}	56.5	17.1	2.5	6.0	7.6	6.6	3.7	1.9	0.9	0.2	0.1	0.1	0.9	2.0	1.7
Inf. target	$\eta^{\overline{\Pi}}$	6.4	2.3	0.5	0.3	0.6	0.8	0.6	0.4	0.2	0.1	0.1	0.1	0.1	0.3	0.2
Consumption tax	η^{τ^C}	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.8	0.0	0.0	1.3
Foreign demand	η^{Y^*}	0.1	2.8	2.4	0.2	1.0	1.8	1.0	4.9	14.0	0.2	1.5	10.2	0.4	6.1	9.0
Foreign supply	η^{Π^*}	0.2	0.5	0.1	0.4	0.4	0.5	1.0	0.8	1.0	0.1	0.2	0.5	0.1	0.2	0.4
Foreign interest rate	η^{R^*}	0.2	0.3	0.2	0.3	0.3	0.4	1.0	1.8	3.2	0.2	0.7	2.0	0.0	0.3	0.9
World trade	η^{WT^*}	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.8	0.4	0.3
Oil price	η^{OIL}	1.1	6.6	1.3	1.5	8.7	11.5	0.0	0.2	0.5	0.6	5.0	2.9	0.1	0.5	2.2
Foreign LR rate	$\eta^{*,fwd}$	0.1	0.9	0.7	0.2	0.4	0.6	0.1	0.2	1.0	0.1	0.4	1.1	0.0	0.1	0.4

Table 7: Forecast Error Variance Decompostion (cont.)

(one-quarter, four-quarter and infinite horizon)

		Output $\hat{y}$			Consumption $\hat{c}$			Investment $\hat{i}$			Exports $\hat{x}$			Imports $\hat{im}$		
		1	4	∞	1	4	∞	1	4	∞	1	4	∞	1	4	∞
Transitory techn.	η	6.5	28.7	36.4	1.4	3.5	9.4	1.5	2.8	7.0	0.5	3.8	6.2	0.8	1.6	2.2
Permanent techn.	η^{g_z}	0.0	0.1	1.1	0.2	0.3	3.3	0.5	1.1	1.8	0.0	0.0	0.2	0.0	0.1	0.2
Symmetric prem.	η^{RP}	11.0	9.3	5.7	55.9	55.6	34.8	33.6	28.4	20.8	0.2	1.5	4.3	14.2	23.4	22.5
External prem.	η^{RP^*}	2.3	2.7	1.6	1.9	4.2	3.6	2.0	2.0	2.5	1.2	2.6	2.1	6.0	6.4	5.0
Dom. prem.	η^{DRP}	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumption	η^{C}	3.3	3.3	2.0	36.4	28.7	16.5	0.4	0.7	1.9	0.0	0.1	0.1	2.2	2.8	2.1
Inv. techn.	η^{I}	3.2	4.8	18.7	0.5	1.0	8.9	58.2	58.6	41.0	0.0	0.0	3.8	2.2	2.3	2.6
Inventory inv.	$\eta^{\Delta INV}$	22.4	8.5	4.4	0.4	0.9	2.0	0.4	0.7	1.6	0.0	0.0	0.1	17.9	7.4	4.7
Government	η^{G}	0.8	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Export share	η^{ν^*}	16.6	11.8	6.4	0.0	0.2	3.5	0.3	0.8	4.0	76.0	42.4	23.5	13.0	18.3	16.9
Home bias	η^{ν}	23.6	9.9	5.1	0.0	0.1	1.7	0.1	0.4	1.9	0.1	0.9	2.1	33.3	13.7	8.3
Domestic markup	η^{H}	2.4	3.2	1.6	0.3	0.5	0.3	0.2	0.2	0.1	0.1	0.3	0.2	0.9	0.5	0.4
Import markup	η^{*}	1.3	0.5	0.3	0.5	0.6	0.3	0.3	0.2	0.1	0.1	0.3	0.2	5.6	3.1	1.9
Wage markup	η^{W}	1.4	6.2	5.2	0.1	0.5	1.0	0.2	0.4	0.7	0.1	0.9	1.0	0.4	0.7	0.7
Export markup	η^{X}	1.6	1.8	0.9	0.0	0.1	0.1	0.0	0.0	0.0	8.6	7.8	4.4	1.0	1.5	0.9
Foreign transfers	η^{FTR}	0.0	0.0	0.6	0.3	0.7	2.7	0.5	0.9	2.7	0.0	0.2	0.6	0.2	0.8	2.1
Interest rate	η^{R}	1.4	1.8	0.9	1.2	0.7	0.4	0.3	0.2	0.1	0.1	0.2	0.1	0.0	0.0	0.1
Inf. target	$\eta^{\overline{\Pi}}$	0.2	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumption tax	η^{τ^C}	0.0	0.0	0.2	0.0	0.0	0.7	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
Foreign demand	η^{Y^*}	0.6	5.4	5.5	0.1	0.3	6.7	0.3	0.3	8.5	7.0	31.8	37.1	1.1	8.1	18.1
Foreign supply	η^{Π^*}	0.2	0.2	0.2	0.0	0.1	0.2	0.0	0.0	0.2	0.4	0.2	1.2	0.0	0.2	0.9
Foreign interest rate	η^{R^*}	0.0	0.2	0.6	0.0	0.0	0.9	0.0	0.0	1.0	0.2	2.2	4.4	0.3	1.7	3.7
World trade	η^{WT^*}	1.2	0.3	0.2	0.0	0.0	0.0	0.0	0.0	0.0	4.8	0.9	0.4	0.5	0.2	0.1
Oil price	η^{OIL}	0.1	0.5	1.2	0.2	1.1	1.0	0.4	0.6	0.7	0.2	2.3	4.3	0.4	7.2	5.8
Foreign LR rate	$\eta^{*,fwd}$	0.0	0.1	0.8	0.3	0.7	1.9	0.7	1.5	3.1	0.2	1.6	3.5	0.1	0.1	0.7

The main shocks contributing to the variation of output ($\hat{y}$) in the short run (cf. the column for the one-quarter horizon in Table 7) are demand shocks: the home bias shock (24%), the shock to the change in inventories (22%), the export share shock, which affects the demand for Israeli exports (17%), and the symmetric premium shock which affects domestic demand (11%). In contrast, the variation in output for longer horizons is attributed in large part to technology shocks (cf. the column for the infinite horizon): the transitory technology shock (36%) and the investment-specific technology shock (19%). The latter affects output through its effect on the demand for investment, and also through the effect of the capital stock on the supply of output.

Demand shocks are the main contributors to the variation of imports ($\widehat{im}$). In the short run, the home-bias shock, which affects the import intensity in the production of final goods, is responsible for 33% of the variation in imports, the inventories demand shock for 18%, the symmetric premium shock for 14% and the export share shock for 13%. Inventories have high short-run volatility as well as high import intensity. The symmetric premium shock affects domestic demand, part of which is satisfied by imports. It also affects the demand for imports through its effect on the prices of domestic goods as well as the prices of imported goods (through its effect on the exchange rate). At the infinite horizon, 22.5% of the variation of imports is attributed to the symmetric premium shock, while the foreign demand shock and export share shock, both of which affect the demand for exports, contribute 18% and 17%, respectively. An increase in the demand for exports raises imports through two channels: First, there is a significant import intensity in the production of exports and second, a rise in net exports leads to an appreciation of the exchange rate (through the external risk premium), thereby lowering the price of imports relative to the price of domestic output.[48] The variance decomposition of the real exchange rate ($\hat{s}$) in the long run reflects the close (two-sided) relationship between the

[48]The foreign demand shock also affects the demand for imports through its effect on foreign prices and therefore on the prices of imports.

real exchange rate and foreign trade (exports and imports). Thus, out of the five shocks whose contribution to the real exchange rate's variance in the long run is greater than 10%, three (i.e. the symmetric premium shock, the export share shock and the foreign demand shock) are also the dominant shocks in the variation of exports and imports. The other two are the transitory technology shock (19%), which affects the real exchange rate through its effects on domestic prices ($p_{H,t}$) and on the nominal exchange rate (see the corresponding impulse response functions in section 6.2) and the external risk premium shock (13%), which directly affects the nominal exchange rate. The external risk premium shock is dominant in the short run, with a 56% contribution to the one-quarter forecast error variance.

The main shocks contributing to the long-run variation of the labor market variables (the real wage, $\hat{w}$, and hours worked, $\hat{N}$) are the transitory technology shock (10.0% and 20.2%, respectively), the symmetric premium shock (11.1% and 8.0%), the export share shock (8.2% and 11.2%), the foreign demand shock (10.2% and 9.0%) and the wage markup shock (24.2% and 9.4%). All but the last affect the labor market by shifting the demand for labor. The wage markup shock may be thought of as a shift in the supply of labor. For the one-quarter horizon, the wage markup shock is dominant in accounting for the variation of real wages (with a contribution of 66%), whereas the transitory technological shock accounts for 38% of the variability of hours worked.

In the case of the nominal interest rate (r^{OB}), 75% of its infinite-horizon variance is due to the highly persistent domestic risk premium shock, which determines the time-varying long-run real interest rate. The dominance of this shock stems from our assumption that it is characterized by a nearly unit root (AR coefficient of 0.99). However, for the shorter horizon its contribution to variance is negligible. The dominant shocks for the one-quarter horizon are the interest rate shock (56%), which may be interpreted as a deviation from the policy rule, and the external risk premium shock (26%), which affects the interest rate through its response to both inflation and a nominal depreciation.

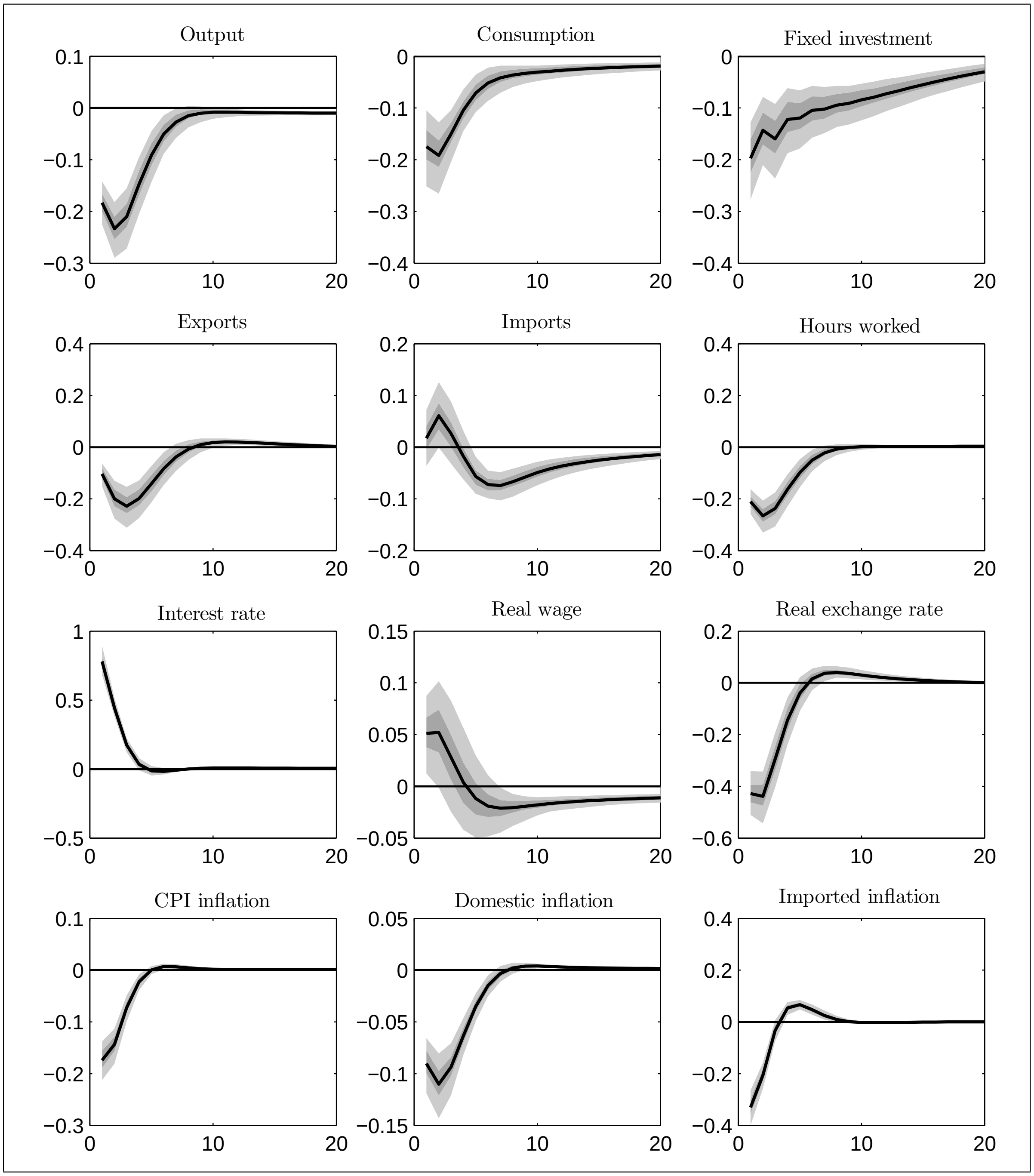
6.2 Impulse response functions

Figures 6 to 10 present the impulse response functions (IRFs) for several key variables following five types of shocks:[49] a monetary policy shock (η^R), an external risk premium shock (η^{RP^*}), a symmetric premium shock (η^{RP}), a foreign demand shock (η^{Y^*}) and a transitory technology shock (η). The IRFs are presented with Bayesian intervals constructed from the posterior distribution, which reflect the uncertainty with respect to both the size (i.e. standard deviation) of the shocks and the parameters. The former is addressed by using a distribution of shocks, rather than just a single shock, which is based on the posterior distribution of the shock's standard deviation, while the latter is addressed by setting the IRFs distributions to correspond with the posterior simulation-based distribution of the parameters. The IRFs are thus computed for each draw from the posterior simulations discussed in section 4.5. Each figure presents the mean of the response and the 70 and 90 percent highest probability intervals. All real variables are expressed as percentage deviations from the model's steady state; the inflation measures are presented as percentage point deviations and the interest rates are presented as annualized percentage point deviations.

Figure 6 presents the impulse responses following a monetary policy shock. As can be seen, an innovation of one standard deviation to the interest rate rule (62) triggers an immediate rise in the interest rate of 0.75 percentage points. Due to the nominal frictions in the model (such as price and wage stickiness), the real interest rate rises as well, leading to a reduction in domestic demand (i.e. consumption and investment) that persists for about two years. The rise in the interest rate also brings about an appreciation of the domestic currency. Consequently, monopolistic exporters gradually raise their foreign currency prices, thereby reducing the demand for their products. As a result, exports fall to 0.2 percent below the steady-state level. Import demand is affected by two opposing forces: the reduction in domestic demand, which reduces the demand for imported interme-

[49] Additional shocks not discussed in the text are presented in figures 11 to 18.

Figure 6: Impulse Response to an Interest Rate Shock



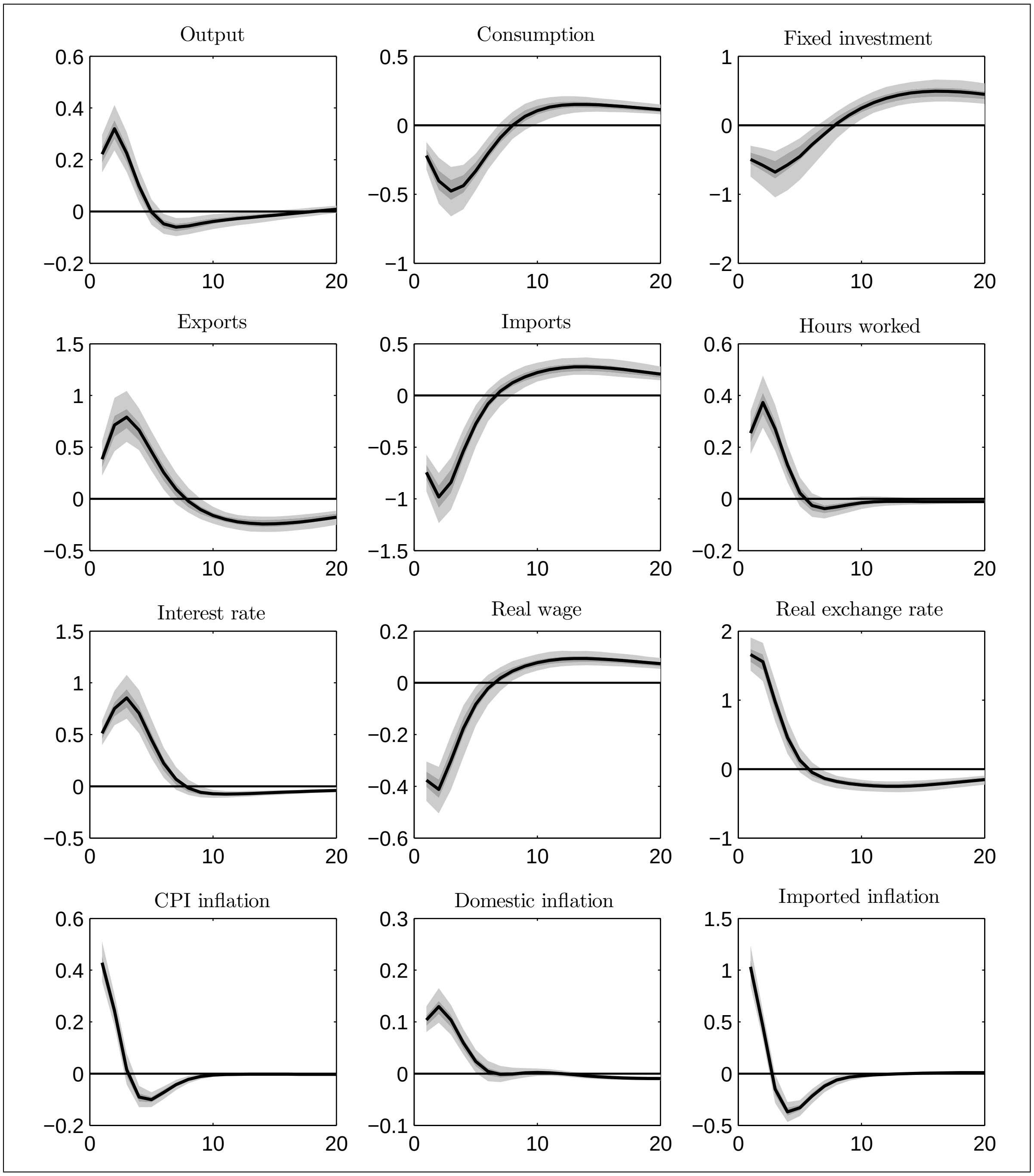
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

diate goods, and the appreciation in the domestic currency, which generates an expenditure switching effect. The results suggest that the expenditure switching effect is in most cases dominant in the short run. In the case of output, both the effects mentioned above, i.e. the contraction in domestic demand and the expenditure switching effect, operate in the same direction to reduce domestic activity and therefore output falls by approximately 0.2 percent. Note that output reaches its lowest level only after two quarters and gradually converges back to its trend within two years. Inflation falls immediately and the accumulated effect after one year is about 0.4 percentage points. Interestingly, the drop in inflation results from both the direct effect of the appreciation on imported inflation and lower marginal costs (wages and capital rental rates). Note that marginal costs fall not only as a result of the contraction in economic activity, but also through the appreciation's effect on wage demands. Taking into account the intensity of the effect on each inflation component and the weight of each component in consumption, domestic and imported inflation make similar contributions to the reduction in the CPI rate of inflation.

It is interesting to compare our model's impulse responses to those reported for similar models, such as Christoffel et al. (2008) and Adolfson et al. (2007) for the euro area, Adolfson et al. (2008) for Sweden, and Beneš et al. (2009) for New Zealand. The comparison leads to four general observations: (1) The size of the shock in our model is typically larger by a scale of 1.5 to 3 (reflecting a larger estimated standard deviation of the interest rate shock). (2) While the effect on output in our model is typically smaller, mainly due to the lower sensitivity of investment, the effect on inflation is larger due to a more rapid exchange rate passthrough (combined with a high import intensity). (3) The reaction of output in our model is more rapid and less hump shaped. Thus, in other economies, the strongest effect on output is typically three to four quarters following the shock, compared to only two quarters in our model. (4) The duration of the shock's effect is only two years in our model, as compared to five years in other economies.

Figure 7 presents the impulse responses following an exogenous shock to the external

Figure 7: Impulse Response to an External Risk Premium Shock

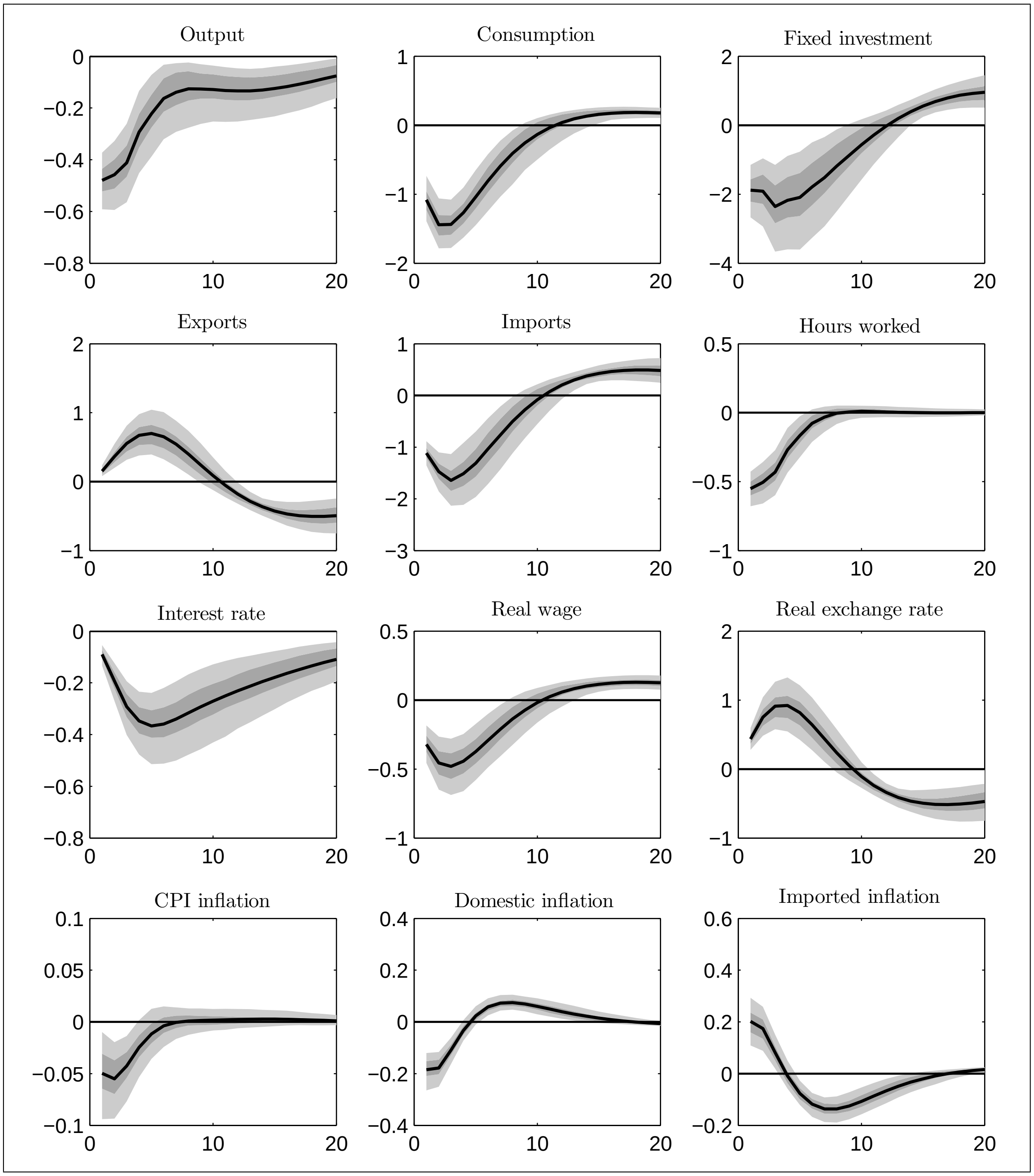


Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

risk premium ($\eta_t^{RP^*}$), which is typically referred to as an exchange rate shock or UIP shock (see section 3.1). The increase in the external risk premium makes the holding of domestic currency bonds less attractive relative to holding foreign currency bonds. The restoration of equality between expected returns is achieved through an immediate nominal depreciation of the domestic currency (2.3 percent), along with a rise in the domestic interest rate. The depreciation in the exchange rate directly raises CPI inflation through the price of imported intermediate goods (the contemporaneous elasticity of the CPI with respect to a depreciation is 0.18). The real depreciation enhances the current account surplus since exports rise by approximately 0.5 percent while imports fall (i.e. the expenditure switching effect dominates the direct effect of the depreciation on the current account). Although domestic demand components fall, due to the rise in the real interest rate and the increased cost of imports, output expands by approximately 0.3 percent, owing to the expenditure switching effect. The increases in inflation and output, as well as the local currency depreciation, leads to a hike in the interest rate. Thus, the nominal interest rate rises by 0.5 percentage points on impact and up to 1.0 percentage points within three quarters. The depreciation is rather short lived and the real exchange rate returns to its original level within two years. Note that domestic inflation, and not only imported inflation, rises following the external risk premium shock, though to a much lesser extent. This is due to the increase in marginal cost, which is the result of four factors: (1) the expansion in economic activity, which influences marginal productivity and the cost the of production inputs; (2) the effect of a depreciation on requested wages and capital rental rates, which is due to their initial drop with respect to the CPI (which includes import prices); (3) the decline in investment which results in a shortage of capital, thereby further raising the rental rate; and (4) the increase in the nominal interest rate, which raises wage bill costs also through the working capital channel (see section 3.2.1).

Figure 8 presents the impulse responses following a symmetric risk premium shock. A positive shock increases the attractiveness of financial risk-free assets (both domestic and

Figure 8: Impulse Response to a Symmetric Premium Shock

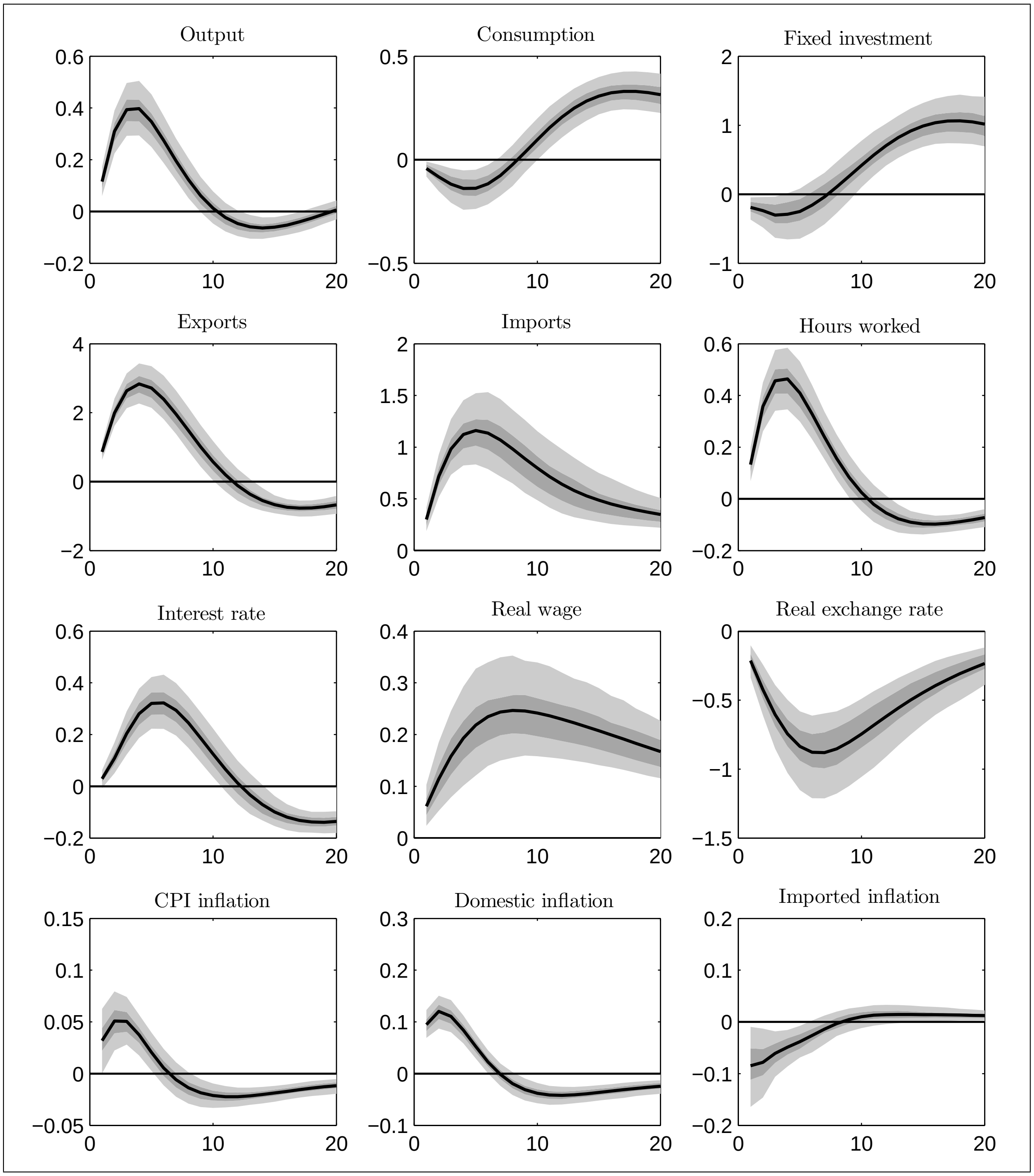


Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

foreign currency bonds) relative to real uses and therefore can be interpreted as a domestic demand shock. Indeed, consumption and fixed capital investment fall on impact by about one and two percent, respectively. The reduction in domestic final uses is transmitted to the demand for output (which declines by approximately 0.5%) and imports (which decline by approximately 1.5%). Imports fall more than output due to an expenditure switching effect which will be discussed below. The resulting excess capacity lowers domestic costs and reduces inflation (by 0.2 percentage points during the first year). The central bank responds to lower inflation and output by gradually reducing the interest rate. The resulting depreciation of the local currency generates an expenditure switching effect, which explains the above-mentioned import-biased contraction, and also a gradual increase of up to 0.7 percent in exports. The duration of the shock's effect is approximately ten quarters.

Figure 9 presents the impulse responses following a foreign demand shock. A positive shock shifts the foreign IS curve (equation 89) upward, leading to an expansion of 0.6 percent in foreign output. As shown by the forecast error variance decomposition (section 6.1), it is the most important foreign shock in the explanation of the domestic real economy. There are various transmission mechanisms through which it operates: First, it directly leads to an expansion of two percent in world trade (see equation 91), thus increasing the demand for exports, which rises by approximately two percent. In addition, the increase in output generates inflationary pressures in the foreign economy (see equation 92), which raise the marginal costs of importers (see equation 30). The expansion of foreign output and the rise in inflation trigger a hike of 0.5 percentage points in the foreign interest rate within a year (see equation 94). The initial widening of the differential between foreign and domestic interest rates creates pressure for a depreciation in the domestic currency. However, figure 9 shows that the real exchange rate appreciates, which is due to the offsetting effects of the current account surplus, which raises net foreign assets and reduces the external risk premium. The net effect is higher domestic output (approximately 0.4 percent at the peak), higher inflation (0.2 percentage points in the first year) and a tightening of monetary policy

Figure 9: Impulse Response to a Foreign Demand Shock

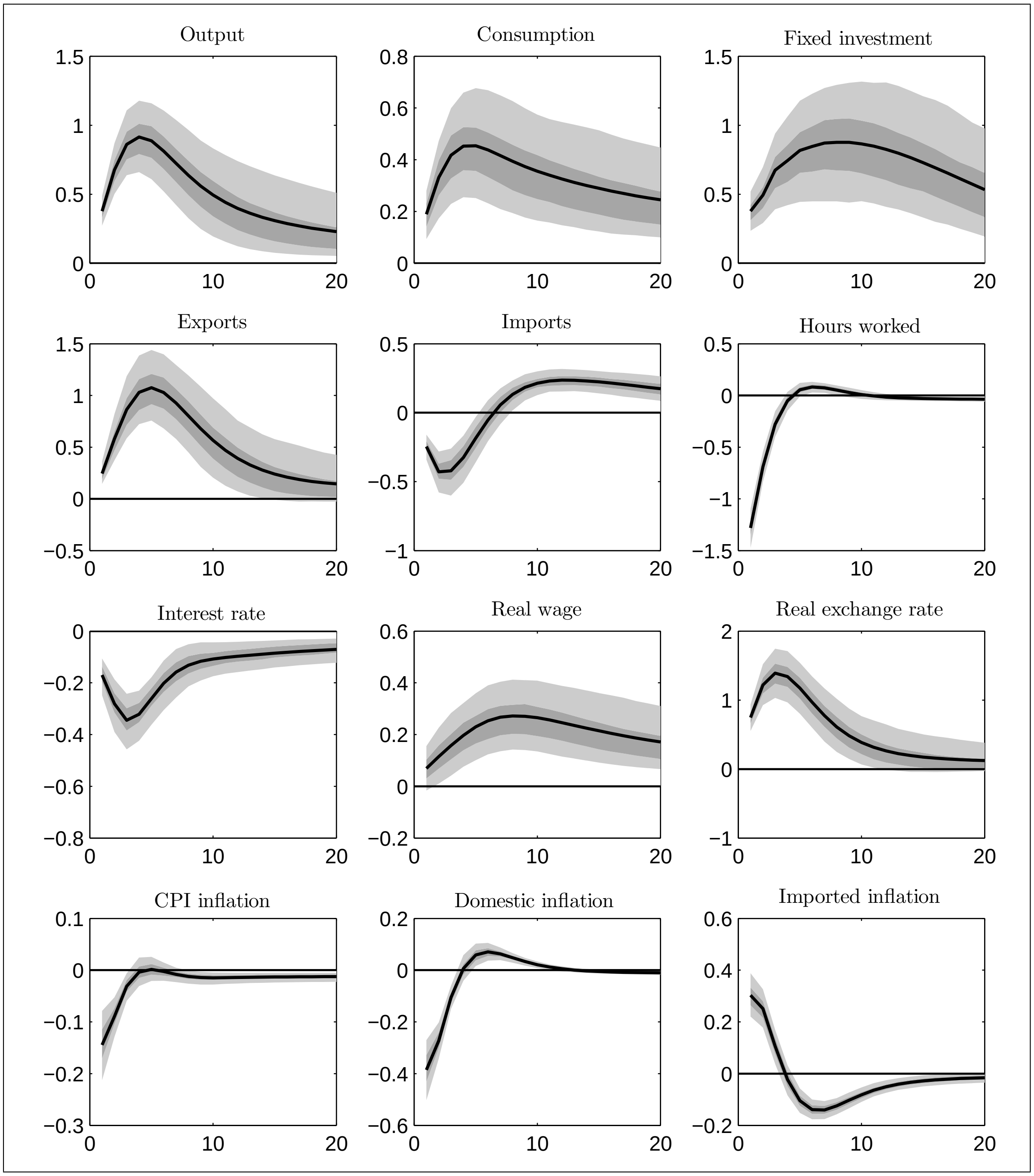


Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

(an interest rate hike of approximately 0.3 percentage points within a year). Note that the tightening of monetary policy results in an initial drop in consumption and investment. However, a positive income effect becomes dominant after approximately two years, with consumption and investment rising to above their steady-state levels.

Figure 10 presents the impulse responses following a transitory technology shock, i.e. a transitory rise in total factor productivity, ε_t (see equation 17). In this case, it is convenient to distinguish between short- and medium-run dynamics. The short run can be interpreted as a period of excess supply and the medium run as a period of excess demand. The transitory shock peaks on impact and then gradually dies out. In the short run, for any given level of production inputs, more output is produced. However, given the model's nominal frictions and the existence of a monetary authority that intervenes in the credit market, output is demand-determined. Although aggregate demand increases as well (through channels that are described below), it does not increase as much as the direct effect of the shock on output. Therefore, and since the capital stock is predetermined, it follows that capital's rental cost and hours worked fall in the short run or, put differently, labor is replaced by technology. Marginal cost falls not only as a result of the direct effect of the shock on marginal productivity, but also as a result of the lower capital rental cost. In addition, the decrease in hours worked further increases marginal labor productivity. (All these effects, which work to reduce marginal cost, dominate the increase in the real wage, which results from the income effect on labor supply and from nominal wage rigidity.) The lower marginal cost reduces DPI inflation and is followed by CPI deflation as well (CPI inflation in the first year is reduced by 0.3 percentage points). This generates a real depreciation which is enhanced by a nominal one that follows the adoption of a more expansionary monetary policy (the real exchange rate peaks at 1.4 percent). The resulting expenditure switching effect helps to match the demand and supply of domestic output. There are additional forces driving the increase in domestic demand for consumption and investment, thus helping to restore general equilibrium. Thus, consumption increases not

Figure 10: Impulse Response to a Transitory Technology Shock



Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

only as a result of the income effect but also as a result of the decline in the real interest rate.[50] Investment is particularly forward-looking due to its adjustment cost and the time needed to build up the capital stock. Therefore, although in the very short run the real interest rate increases and capital rental cost falls, Tobin's Q increases right away, thus accounting for the expectations of a lower real interest rate and a higher capital rental cost in the future. Overall, there is growth in both domestic supply and demand during the first few quarters following the shock, but the supply of domestic output increases by more than aggregate domestic demand and therefore it can be viewed as a period of excess domestic supply, which is reflected by higher exports and lower imports.

The medium run, as mentioned above, can be characterized as a period of excess demand, since the effect of the shock begins fading away while demand remains high as described above. As a result, production costs increase, as does DPI inflation. However, the accumulated current account surpluses (due to the aforementioned expenditure switching effect) lead to a nominal appreciation that reduces imported inflation. As a result, CPI inflation remains low for a while. The excess demand in the medium run leads to an increase in imports, which is supported by the current account surpluses in the first few quarters following the shock. It is thus interesting to note how the combination of market forces and policy leads to an increase in utility long after the initial shock has faded away, as reflected by the increases in consumption and leisure. The current account acts as a buffer which facilitates the apparent discrepancy between the rapid convergence of output and the smooth long-lasting effect on utility. The persistence of the shock is relatively high (0.86). Together with the mechanisms described and the manner in which they prolong the impact of the shock, its expansionary effect on some variables lasts beyond five years.

[50] The real interest rate increases in the short run due to the drop in expected inflation, which dominates the effect of expansionary monetary policy, but declines from the second quarter onward.

7 Concluding remarks

In this paper, we have presented a medium-scale micro-founded DSGE model developed to support monetary policy formulation and risk assessment at the Bank of Israel. This type of model is commonly used by central banks worldwide. It was estimated using standard Bayesian techniques applied to quarterly Israeli data for the period 1992-2009. The estimation results are for the most part satisfactory: most parameters are well-identified, the stochastic model replicates most of the observed moments that characterize key macro variables and its (in-sample) unconditional forecasting quality is comparable with that of both naive and statistical models.

In formulating the model's structure, we built on earlier work done by other central banks and made some modifications in order to better capture important features of the Israeli economy and better equip the model for practical use. The modifications include: the addition of imports as an intermediate good in the production of exports, extensions addressing the time-varying nature of long-term real interest rates during the sample period, the disinflation process during the early years of the sample period, the adjustment of the model's aggregate resource constraint to satisfy national accounts identities through the introduction of inventories etc.

Most of the modifications are highly simplified, but nevertheless were found to improve the overall fit of the model and enhance its usefulness in supporting monetary policy. On the other hand, simplification comes at a cost. Thus, some moments in the data are not replicated well, partly because of the simplicity of the modifications. For instance, the model fails to replicate the auto-correlation and standard deviation of output growth, partly due to the exogenous process assumed for investment in inventories. In addition, the model fails to replicate the negative correlation between the interest rate and lagged inflation, which we do not have a good explanation for. One possibility is a misspecified (ad hoc) interest rate rule; however, it is worth noting that the model forecasts the interest rate quite well, and the estimation of other structural parameters is robust to alternative specifications of the interest rate rule.

Another important modification was meant to address the challenge of taking a theoretical cyclical model with long-run balanced growth to data characterized by imbalanced growth. For this purpose, we enhanced the observation equations with stochastic components in an attempt to capture imbalanced growth rates, thus avoiding any pre-filtering and loss of relevant information embedded in the co-movement of the variables in the data.

The estimation results, and the corresponding properties of the model, point to some duality between real and nominal variables. This is best illustrated by the variance decomposition which shows a clear distinction between the main shocks accounting for the variance of real variables and those accounting for the variance of nominal variables. Moreover, similar models for other countries suggest that in transmission from monetary policy shocks to inflation, the channel of domestic price inflation is stronger than that of imported inflation, although they do work in the same direction. In contrast, we found both channels to be equally strong. An additional, though related finding concerns the speed of transmission in Israel. Thus, the response of inflation to a monetary policy shock peaks on impact and the response of output peaks in the following quarter. However, the effect is also short-lived relative to typical findings in other countries. The speed and short duration of transmission from monetary policy are partly a result of the relatively weak frictions, both real and nominal, estimated for the Israeli economy. The average duration between reoptimization of prices in the various sectors of the economy varies between 1.8 to 2.8 quarters, which is lower than the estimates for other countries (which typically range between 3-4 quarters).

The purpose of the paper has been to present the model, evaluate its empirical fit and discuss its main properties. In future work we hope to present some applications of the model and to further extend it. Thus, in addition to using the model for forecasting and risk assessment, it is used to address various research questions. In particular, the model is useful in evaluating the actual conduct of monetary policy using counterfactual and other types of simulation. In addition, we are considering enriching the model's labor and financial markets, a direction taken by other central banks following the recent economic developments worldwide.

Appendices

Appendix A The data

The data consists of 24 macroeconomic time series for the sample period 1992:Q1 - 2009:Q4. All the data series were seasonally adjusted, except for the interest rates, the exchange rate, tax rates and the price of oil. All variables are expressed in terms of log differences, i.e. $\Delta X_t \equiv \log\left(x_t/x_{t-1}\right)$, unless otherwise stated. Variables that are expressed in per capita terms are divided by the size of the working age population. Following is a description of the observed variables used in the estimation, by category:

Real National Accounts data

This group includes the main components of the national accounts balance sheet. Some subcomponents, which are characterized by low added value and high volatility that is not related to macroeconomic conditions, were excluded. Data satisfying the standard of the System of National Accounts (SNA93) was only available starting from the first quarter of 1995; for the preceding period, the data is based on the previous standard. Following are the variables in this category (all are expressed in per capita terms and in constant prices):

- Output, ΔY_t - gross domestic product.
- Private consumption, ΔC_t - private consumption expenditure.
- Investment, ΔI_t - gross fixed capital formation excluding ships and aircraft.
- Government consumption, ΔG_t - general government consumption, excluding defence imports.
- Exports, ΔX_t - exports of goods and services, excluding diamonds and start-up companies.

- Imports, ΔIM_t - imports of goods and services, excluding defence imports, ships, aircraft and diamonds.

Source: Central Bureau of Statistics (CBS). All data in this group was seasonally adjusted by the CBS.

Prices and Inflation

- CPI, $\Delta P_{C,t}$ - Consumer Price Index. Source: CBS. The fruit and vegetable component (approximately 3% of the CPI) was excluded since it is poorly explained and characterized by high volatility. Due to the widespread dollarization in the housing sector prior to 2007 (90% of rental contracts were denominated in dollars), housing was also excluded from 1992 until 2006. Since dollarization weakened in the second part of the sample (with 90% of the contracts now denominated in NIS terms) the CPI data excludes only fruits and vegetables from 2007 onwards. In addition, since the model's CPI inflation ($\hat{\pi}_{C,t}$) is expressed in factor prices (before indirect taxes) whereas the CPI data is expressed in market prices, we deducted the changes in the VAT rate from observed CPI inflation.[51]

- Output price, $\Delta P^{M}_{Y,t}$ - GDP deflator in market prices. Source: CBS national accounts data.[52]

- Export price in domestic currency, $\Delta P^{NIS}_{X,t} = \Delta\left(S_t P_{X,t}\right)$ - Deflator of exports excluding diamonds and start-up companies (in market prices). Source: CBS National Accounts data.

- Annualized inflation target, $4 \cdot \overline{\pi}_t$. Source: Bank of Israel.

[51] Using the appropriate, time-varying weight. The weight of the CPI components subject to VAT varies during the sample period, since housing rents are not subject to VAT.

[52] The observation equation connecting this variable to the model contains measurement error.

The inflation target was subtracted from all the above nominal variables, so as to achieve stationarity. This was followed by seasonal adjustment.

Interest rates and the exchange rate

- Nominal interest rate, r_t^{OB} - Annualized key nominal interest rate set by the Bank of Israel.
- 5-10 year forward real interest rate, $rr_t^{fwd,OB}$ - The 5-10 years forward real interest rates, which are derived from indexed government bond yields.
- Nominal exchange rate, ΔS_t - Israel's weighted nominal effective exchange rate against the currencies of its four major trading partners. The weights are as follows: US dollar - 49%, Euro - 32%, Sterling - 13%, Yen - 6%. These weights are based on an OLS regression of the nominal effective exchange rate (consisting of more than 20 currencies) on the four selected currencies. An increase in the value of this variable indicates a depreciation.

The data for interest rates and exchange rates is not seasonally adjusted. The inflation target was subtracted from the nominal interest rate and from the change in the exchange rate. Source: Bank of Israel.

Labor market data

- Nominal hourly wage, ΔW_t - total wages divided by total hours worked. The inflation target was subtracted from this variable in order to maintain consistency with the above-mentioned nominal variables.
- Hours worked, ΔN_t - Total hours worked per working-age population.
- Employment, ΔEM_t - Total employees per working-age population.

Source: CBS and the National Insurance Institute. Data has been seasonally adjusted by the CBS.

Foreign data

- G4 nominal interest rate, $r_t^{*,OB}$ - Weighted average of the nominal central bank key interest rates of the G4 countries. Source: Bloomberg.
- G4 CPI, $\Delta P_{Y,t}^*$ - Seasonally adjusted weighted average of the consumer price indices of the G4 countries. Source: OECD database.
- G4 GDP, ΔY_t^* - Weighted average of the gross domestic product of the G4 countries, in fixed prices; seasonally adjusted by G4 agencies. Source: OECD database.
- OECD imports, ΔWT_t^* - Weighted average of the volume of OECD imports of goods and services; seasonally adjusted. Source OECD database.
- 5-10 year forward G4 nominal interest rate, $r_t^{*,fwd,OB}$ - The 5-10 years forward nominal interest rates, extracted from non-indexed government bond yields. Source: Bloomberg.

The weights are equivalent to those estimated for the nominal exchange rate, ΔS_t: USA (49%), Euro area (32%), Britain (13%), Japan (6%).

Miscellaneous data

- VAT rate, τ_t^C - VAT rate. Source: Ministry of Finance.
- Current account surplus, $s_{CA,t} \equiv CA_t/P_{Y,t}^M Y_t$ - Seasonally adjusted ratio of the current account surplus to nominal GDP. Source: CBS.
- Oil price, $\Delta P_{OIL,t}^*$ - Price per barrel of Brent crude oil in terms of the effective exchange rate. Source: Bloomberg.

Appendix B Model-consistent filtering

Real variables in the theoretical model are characterized by balanced growth. Thus, all real trends share the growth rate of the labor-augmenting technology shock, $\hat{g}_{z,t}$ in the linear version of the model (see equation 18). The data, however, is characterized by imbalanced growth (see figure 2 and section 4.2). Therefore, each observed variable has a so-called excess trend that needs to be filtered out. This appendix describes how the excess growth components of each variable were handled in a model-consistent manner.

B.1 Detrending employment variables

In order to remove the excess trends from the observed employment variables (and consequently from the real domestic variables), we defined a secular growth component in hours worked (GR_t^N) and assumed that it follows an auto-regressive process:

$$GR_t^N = \left(1 - \rho_{GR}^N\right) g_{\Delta N} + \rho_{GR}^N GR_{t-1}^N + \eta_{GR,t}^N, \tag{98}$$

where $g_{\Delta N}$ is the (gross) long-run growth rate of per capita hours worked (and of the employment rate), ρ_{GR}^N is the rate of first-order auto-correlation in the rate of secular growth of hours worked (to be estimated within the model) and $\eta_{GR,t}^N$ is an *i.i.d.* innovation to the rate of secular growth.[53]

Given (98), following are the relevant observation equations that link observed hours, employment and (per capita) output $\left(\Delta N_t^{OB}, \Delta EM_t^{OB} \text{ and } \Delta Y_t^{OB}\right)$ to the unobserved deviations of the (stationarized) variables from their steady states ($\hat{N}_t$, $\widehat{EM}_t$ and $\hat{y}_t$):[54]

$$\Delta N_t^{OB} = \hat{N}_t - \hat{N}_{t-1} + GR_t^N, \tag{99}$$

[53] Strictly speaking, since per capita hours worked are bounded, their long-run growth rate must be zero. We essentially assume that the AR processes for the excess growth rates hold in the sample period and refer to the constant components of these processes as long-run growth rates.

[54] Recall that in the stationarization of the model, real domestic variables are divided by the permanent technological shock z_t, i.e. $\hat{y}_t = y_t - y$, where $y_t = \log(Y_t/z_t)$, and y is the steady state value of Y_t/z_t.

$$\Delta EM_t^{OB} = \widehat{EM}_t - \widehat{EM}_{t-1} + GR_t^N \tag{100}$$

and

$$\Delta Y_t^{OB} = \hat{y}_t - \hat{y}_{t-1} + \hat{g}_{z,t} + (g_z - 1) + GR_t^N. \tag{101}$$

We added the superscript OB in order to distinguish between the observed variables (which include the excess trends) from their model counterparts.

B.2 Detrending aggregate quantities of goods

We now turn to the removal of the idiosyncratic excess growth paths from private consumption (C), fixed capital investment (I), government consumption (G), exports (X), imports (IM), foreign GDP (Y^*) and world trade (WT^*). For each component $J \in \{C, I, X, IM, Y^*, WT^*\}$, we specify the following auto-regressive process for the excess trend (EX_t^J):

$$EX_t^J = \left(1 - \rho_{EX}^J\right)\left(g_{\Delta J} - g_z - j^H(J) \cdot g_{\Delta N}\right) + \rho_{EX}^J EX_{t-1}^J + \eta_{EX,t}^J\ , \tag{102}$$

where $j^H(J)$ is an index function for domestic components, which is used to account for the fact that the secular growth in hours worked is only relevant for domestic variables:

$$j^H(J) = \begin{cases} 1 & if \quad J \in \{C, I, X, IM\} \\ 0 & if \quad J \in \{Y^*, WT^*\} \end{cases} \quad .$$

In (102), $g_{\Delta J}$ is the assumed long-run growth rate of component J. Hence, $g_{\Delta J} - g_z - j^H(J) \cdot g_{\Delta N}$ is the long-run (idiosyncratic) excess growth of component J,[55] ρ_{EX}^J is the first-order auto-correlation in J's excess growth (to be estimated within the model) and $\eta_{EX,t}^J$ is an $i.i.d.$ innovation in the rate of its excess growth. In order to ensure that the long-run aggregate resource constraint on the observed variables is fulfilled, the excess growth in

[55] In the case of the foreign variables, long-run excess growth refers to the deviation of a variable's growth rate beyond its long-run growth rate in the model (g_z) whereas in the case of the domestic components, the idiosyncratic excess growth is the deviation of the growth rate beyond the long-run growth rate of observed GDP, i.e. $g_z + g_{\Delta N} - 1$.

government consumption evolves according to the following condition:

$$s_C EX_t^C + s_I EX_t^I + s_G EX_t^G + s_X EX_t^X - s_{IM} EX_t^{IM} = 0, \tag{103}$$

where s_J is the share of component J in GDP.

Given (98), (102) and (103), the relevant observation equation for each real component $J \in \{C, I, G, X, IM, Y^*, WT^*\}$ that links the observed variable ΔJ_t^{OB} to the unobserved (log) deviation of J_t/z_t from its steady state (denoted by $\hat{j}_t$) is given by:

$$\Delta J_t^{OB} = \hat{j}_t - \hat{j}_{t-1} + \hat{g}_{z,t} + (g_z - 1) + j^H \cdot GR_t^N + EX_t^J. \tag{104}$$

B.3 Detrending relative prices

Excess trends are also removed (in a similar model-consistent manner) from the following relative prices: the real exchange rate, real hourly wages and the relative price of oil. The corresponding observation equations are:

$$\Delta S_t^{OB} = \hat{s}_t - \hat{s}_{t-1} + \left(\hat{\pi}_{Y,t} + \bar{\Pi} - 1\right) - \left(\hat{\pi}^*_{Y,t} + \bar{\Pi}^* - 1\right) - \widehat{\bar{\pi}}_t + EX_t^S, \tag{105}$$

$$\Delta W_t^{OB} = \hat{w}_t + \hat{w}_{t-1} + \left(\hat{\pi}_{C,t} + \bar{\Pi} - 1\right) + \left(\hat{g}_{z,t} + g_z - 1\right) - \widehat{\bar{\pi}}_t + EX_t^W \tag{106}$$

and

$$\Delta P_{OIL,t}^{*,OB} = \hat{p}^*_{OIL,t} - \hat{p}^*_{OIL,t-1} + \Delta P^*_{Y,t} + EX_t^{P^*_{OIL}}. \tag{107}$$

Equation (105) relates the observable nominal depreciation rate (ΔS_t^{OB}) to the deviation of the real exchange rate from steady state ($\hat{s}_t$, where $s_t = S_t \cdot P^*_{Y,t}/P_{Y,t}$). The deduction of the inflation target deviation from the right hand sides of (105) and (106) parallels its deduction from the observed nominal variables, as described in section 4.1. We specify EX_t^S as an i.i.d. innovation, rather than an AR process, in order to avoid long-lived gaps between observed changes in the exchange rate and those reflected by the model.

Equation (106) relates observable nominal hourly wage growth (ΔW_t^{OB}) to the (log) deviation from steady state of the scaled real wage ($\hat{w}_t$, where $w_t = \frac{W_t}{P_{C,t} z_t}$). We specify EX_t^W to be an *i.i.d.* innovation, rather than an AR process, in order to filter out noise from the wage data rather than to generate long periods of excess growth.

Equation (107) relates the observable quarterly change in Brent oil prices ($\Delta P_{OIL,t}^{*,OB}$) to the deviation from steady state of the relative price of oil ($\hat{p}_{OIL.t}^*$, where $p_{OIL.t}^* = P_{OIL,t}^*/P_{Y,t}^*$). We specify $EX_t^{P_{OIL}^*}$ to be a first-order auto-regressive process, which is zero in steady state and has a persistence rate of $\rho_{EX}^{P_{OIL}^*}$ (see equation (102)). It can be thought of as reflecting permanent changes in the relative price of oil or simply observation errors.

B.4 Smoothing of forward interest rates

In relating the observed 5-10 years forward interest rates (both domestic and foreign) to the model's expected short-term 5-10 years-ahead interest rates, we allow for a (possibly time-varying) term premium. Thus, we specify the following observation equations for the forward interest rates (the terms in the brackets are multiplied by 4 because the observed interest rate variables are expressed in annualized terms):

$$rr_t^{fwd,OB} = 4 \cdot \left(\frac{g_z}{\beta} - 1 + \widehat{rr}_t^{fwd} \right) + tp + \varepsilon_t^{fwd,OB} \tag{108}$$

and

$$r_t^{*,fwd,OB} = 4 \cdot \left(\frac{g_z}{\beta} \bar{\Pi}^* - 1 + \widehat{rr}_t^{*,fwd} + \widehat{\bar{\pi}}_t^* \right) + tp^* + \varepsilon_t^{*,fwd,OB}, \tag{109}$$

where the term g_z/β is the (gross) real interest rate's steady state. Note that we use nominal forward rates for the foreign economy and assume that agents expect the average foreign 5-10 years-ahead inflation rate to be at its target level ($\widehat{\bar{\pi}}_t^*$). The parameters tp and tp^* (estimated within the model) represent constant term premiums that account for the normal upward slope of the yield curves. We also include serially-correlated observation errors ($\varepsilon_t^{fwd,OB}$ and $\varepsilon_t^{*,fwd,OB}$), which can be thought of as time-varying components of the term premium or simply observation errors. The observed *fwd* rates ($rr_t^{fwd,OB}$ and $r_t^{*,fwd,OB}$) and the smoothed 5-10 years-ahead expected short-run real rates ($\widehat{rr}_t^{fwd}$ and $\widehat{rr}_t^{*,fwd}$) are presented in figure 2.

Appendix C The steady state

This appendix describes the solution of the model's non-stochastic steady-state equilibrium. Section C.1 specifies the steady-state form of the first-order conditions and the market-clearing conditions, which constitute a total of 48 non-linear equations with 48 steady-state unknowns. Section C.2 presents an analytical recursive solution of the steady-state equilibrium conditions, as functions of the model's parameters.

Due to the unit-root technological process in the production of intermediate goods and the unit-root nature of prices, some of the variables in the model are driven by stochastic trends. These variables were normalized in order to render them stationary, which is required for a well-defined steady-state equilibrium. Thus, unless stated otherwise, variables that are driven by a real trend were normalized by the level of productivity, z_t, while those driven by a nominal trend were normalized by the CPI, $P_{C,t}$. In the following equations, the stationarized variables are denoted by lower-case letters, e.g. $c_t \equiv C_t/z_t$ and $p_{H,t} \equiv P_{H,t}/P_{C,t}$. The model produces balanced growth, where all real variables (including world output) grow in the steady state at the same pace, i.e. g_z. Similarly, all steady-state inflation rates are consistent with the inflation target, $\bar{\pi}$.

Note that the steady-state solution is based on the values of certain variables, which are consistent with the model's assumptions:

1. The capital-utilization cost pushes the economy toward full utilization of capital in the non stochastic steady state ($u = 1$).

2. The endogenous foreign exchange risk premium, which is driven by the economy's net foreign assets position, is a stabilization mechanism that pushes the steady-state

net foreign assets position to zero (i.e. $B^* = 0$), which leads to a balanced current account.

3. Due to the model's indexation mechanisms, in the non-stochastic steady-state prices are updated according to the inflation target, regardless of whether one received the Calvo signal. Thus, the solution for the non-stochastic steady state is equivalent to the case of no price rigidity (all $\xi's$ are set equal to zero), which we therefore substitute for simplicity. Similar reasoning applies for the wages as well.

C.1 The non-linear steady-state model

We begin by rewriting the non-linear system as a representation corresponding to the non-stochastic steady state. In other words, all exogenous shocks are cancelled out, which in most cases means they are set to one, and the time subscript t is dropped. In addition, since symmetry is obtained in the non-stochastic steady state, household and firm indices are also dropped.

C.1.1 Households

The first order condition (FOC) with respect to (WRT) C_t (5) in the non-stochastic steady-state becomes:

$$\lambda = \frac{(1-\kappa/g_z)^{-1}}{1+w_{\tau^C}\tau^C}\frac{1}{c}, \tag{110}$$

where $\lambda_t = (\Lambda_t \cdot z_t)$ is the normalized marginal utility of the consumer's income.

FOC WRT B_t (6) in the non-stochastic steady-state becomes:

$$\frac{\beta}{g_z\Pi}R = 1. \tag{111}$$

FOC WRT B_t^* (7) becomes:

$$\frac{\beta}{g_z\Pi^*}R^* = 1, \tag{112}$$

where we assume $\Pi^* = \Pi$.

The physical capital production function (9) in steady state becomes:

$$i = \left(1 - \frac{1-\delta}{g_z}\right) k, \tag{113}$$

where for notational convenience we define $k_t \equiv K_{t+1}/z_t$.

FOC WRT I_t (11) becomes:

$$p_I = Q. \tag{114}$$

FOC WRT K_{t+1} (12) in the non-stochastic steady state, substituting $u = 1$, becomes:

$$Q = \frac{\beta}{g_z} \left[(1-\delta) Q + \left(1 - \tau^K\right) r_K + \tau^K \delta p_I \right].$$

Combining with (114), we obtain:

$$\left[\frac{\beta}{g_z} - \left(1 - \delta + \tau^K \delta\right) \right] p_I = \left(1 - \tau^K\right) r_K. \tag{115}$$

FOC WRT u_t (13), again substituting $u = 1$, becomes:

$$r_K = \gamma_{u,1} p_I.$$

For both this equation and (115) to hold, the following parameter restriction must also hold:

$$\gamma_{u,1} = \left[\frac{\beta}{g_z} - \left(1 - \delta + \tau^K \delta\right) \right] \left(1 - \tau^K\right)^{-1},$$

otherwise the following condition is not satisfied:

$$u = 1. \tag{116}$$

Optimal wage-setting (15) in the non-stochastic steady state, which takes into account all of the system's indexation mechanisms, reduces to:

$$w = \frac{\varphi^W}{\left(1 - \tau^N - \tau^{W_h}\right)} \frac{N^\zeta}{\lambda}, \tag{117}$$

where $w_t = \frac{W_t}{P_{C,t} z_t}$.

C.1.2 Domestic intermediate goods firms

The production function (17) in the steady state becomes:

$$h^s = (g_z)^{-\alpha} (k^s)^{\alpha} N^{1-\alpha} - \psi. \tag{118}$$

Using the definition of the working capital cost, in steady state we obtain:

$$R^F = 1 + \nu^F (R - 1). \tag{119}$$

Cost minimization (24) in the steady state becomes:

$$\frac{k^s}{N} = \frac{\alpha}{(1-\alpha)} g_z \frac{\left(1+\tau^{W_f}\right) R^F w}{r_K}. \tag{120}$$

In the steady state, domestic firms' real marginal cost (25), in terms of consumer prices, is given by:

$$mc = \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} (r_K)^{\alpha} \left[\left(1+\tau^{W_f}\right) R^F w\right]^{1-\alpha}. \tag{121}$$

Optimal price-setting by domestic firms (28) reduces to:

$$\frac{mc}{p_H} = \frac{1}{\varphi^H}. \tag{122}$$

C.1.3 Foreign intermediate goods firms

Taking into account foreign firms' marginal cost (30), the FOC WRT their price (33) reduces to:

$$\frac{s \cdot p_Y \left(p^*_{OIL}\right)^{\omega^*}}{p_{IM}} = \frac{1}{\varphi^*}, \tag{123}$$

where $s_t \equiv P^*_{Y,t} S_t / P_{Y,t}$ is the real exchange rate.

C.1.4 Domestic final goods firms

The steady state demand for domestic intermediate goods (as in 43) in each of the final goods sectors (C, I, G, X) is given by:

$$h^C = \nu_C (p_H)^{-\mu_C} q^C; \tag{124}$$

$$h^I = \nu_I \left(\frac{p_H}{p_I}\right)^{-\mu_I} q^I; \tag{125}$$

$$h^G = \nu_G \left(\frac{p_H}{p_G}\right)^{-\mu_G} q^G; \tag{126}$$

and

$$h^X = \nu_X \left(\frac{p_H}{p_{DX}}\right)^{-\mu_X} q^X. \tag{127}$$

The steady-state demand for foreign intermediate goods (as in 44) in each of the final goods sectors (C, I, G, X) is given by:

$$im^C = (1-\nu_C)(p_{IM})^{-\mu_C} q^C; \tag{128}$$

$$im^I = (1-\nu_I) \left(\frac{p_{IM}}{p_I}\right)^{-\mu_I} q^I; \tag{129}$$

$$im^G = (1-\nu_G) \left(\frac{p_{IM}}{p_G}\right)^{-\mu_G} q^G; \tag{130}$$

and

$$im^X = (1-\nu_X) \left(\frac{p_{IM}}{p_{DX}}\right)^{-\mu_G} q^X. \tag{131}$$

The steady-state competitive prices (as in 45) in each of the final goods sectors (C, I, G, X) are given by:

$$1 = \left[\nu_C (p_H)^{1-\mu_C} + (1-\nu_C)(p_{IM})^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}}; \tag{132}$$

$$p_I = \left[\nu_I \left(p_H\right)^{1-\mu_I} + \left(1-\nu_I\right)\left(p_{IM}\right)^{1-\mu_I}\right]^{\frac{1}{1-\mu_I}}; \tag{133}$$

$$p_G = \left[\nu_G \left(p_H\right)^{1-\mu_G} + \left(1-\nu_G\right)\left(p_{IM}\right)^{1-\mu_G}\right]^{\frac{1}{1-\mu_G}}; \tag{134}$$

and

$$p_{DX} = \left[\nu_X \left(p_H\right)^{1-\mu_X} + \left(1-\nu_X\right)\left(p_{IM}\right)^{1-\mu_X}\right]^{\frac{1}{1-\mu_X}}. \tag{135}$$

C.1.5 Exporters

The exporters' FOC WRT their price (50) reduces to:

$$\frac{p_{DX}}{s p_x p_Y} = \frac{1}{\varphi^X}. \tag{136}$$

C.1.6 Foreign retail firms

The foreign demand for exports (55) is given by:

$$x = \nu^* \left(\frac{p_X}{p_X^{c,*}}\right)^{-\mu^*} wt^* \tilde{z}, \tag{137}$$

where $wt_t^* = WT_t^*/z_t^*$ is scaled world trade and $\tilde{z}_t = z_t^*/z_t$ is the the relative level of technology.

C.1.7 Government

The government exogenously determines the share of government expenditure in GDP (s_G) and the tax rates (τ^C, τ^D, τ^K, τ^N, τ^{W_h} and τ^{W_f}) which will therefore be treated as parameters. The pereiod-by-period budget constraint (58) is given by:

$$\begin{aligned} s_G + \left(\Pi g_z\right)^{-1} s_B \quad = \quad & w_{\tau^C}\tau^C \frac{c}{p_Y y} + \left(\tau^N + \tau^{W_h} + \tau^{W_f}\right)\frac{wN}{p_Y y} \\ & + \frac{\tau^K}{g_z}\frac{r_K k}{p_Y y} - \frac{\tau^K \delta}{g_z}\frac{p_I k}{p_Y y} + \tau^D s_D + s_T + \frac{s_B}{R}. \end{aligned} \tag{138}$$

Since lump sum taxes are driven by the government debt (61), they are zero in steady state:

$$s_T = 0. \tag{139}$$

By definition, real government consumption is given by:

$$g = \frac{p_Y y}{p_G} s_G. \tag{140}$$

C.1.8 Net foreign assets and the current account

The trade balance as a share of GDP is defined by:

$$s_{TB} = s_X - s_{IM}, \tag{141}$$

where the export share is:

$$s_X = \frac{s \cdot p_X x}{y} \tag{142}$$

and the import share is:

$$s_{IM} = \frac{p_{IM} im}{p_Y y}. \tag{143}$$

The existence of an external intermediation premium pushes the steady-state net foreign assets position to zero ($B^* = 0$). Thus, given the dynamics of debt (68), the current account ends up being balanced in the steady state. Therefore, and taking into consideration the definition of the current account (66), we obtain in the steady state:

$$s_{TB} = -s_{FTR}, \tag{144}$$

where s_{FTR} is the exogenous foreign transfer weight in GDP.

C.1.9 Market-clearing conditions

In the steady state, market clearing in the capital market (73) is expressed by:

$$k = k^s. \tag{145}$$

Market clearing in the domestic intermediate goods market (75) implies:

$$h^s = h. \tag{146}$$

In steady state, market clearing in the imported intermediate goods market (77) is given by:

$$im = im^C + im^I + im^G + im^X. \tag{147}$$

Note that there is no equivalent equation for h since it is implied by equations (132-135), (148-151), (147), (152) and (153).

Market clearing conditions in the final goods markets (80-83) are given by:

$$q^C = c, \tag{148}$$

$$q^I = i + \Delta inv \cdot y, \tag{149}$$

where Δinv is the exogenous steady-state weight of inventories investment in GDP;

$$q^G = g, \tag{150}$$

and

$$q^X = x + \psi^X. \tag{151}$$

Nominal GDP (84) in the steady state is given by:

$$p_Y y = p_H h + s \cdot p_X p_Y x - p_{DX} q^X. \tag{152}$$

The aggregate nominal resource constraint (85) in the steady state is:

$$p_Y y = c + p_I i + \Delta inv \cdot p_I y + p_G g + s \cdot p_X p_Y x - p_{IM} im, \tag{153}$$

and when expressed in market prices:

$$p_Y^M y = \left(1 + w_{\tau^C} \tau^C\right) c + p_I i + \Delta inv \cdot p_I y + p_G g + s \cdot p_X p_Y x - p_{IM} im. \tag{154}$$

The definition of real output (87) in the steady state is given by:

$$y = h^s. \tag{155}$$

The steady state share of profits in the export sector is given by:

$$s_{DX} = s \cdot p_X \frac{x}{y} - \frac{p_{DX}\left(x + \psi^X\right)}{p_Y y}, \tag{156}$$

and the economy's total share of profits (88) in steady state is given by:

$$s_D = 1 - \frac{mc}{p_Y}\left(\frac{h^s}{y} + \frac{\psi}{y}\right). \tag{157}$$

C.2 The recursive steady-state solution

The above system of 48 equations (110-157) contains 48 as yet unsolved steady-state endogenous variables. This subsection presents an analytical recursive solution for these endogenous variables, as a function of a sub-set of the model's parameters. we prefer recursive presentation over an explicit presentation of the steady-state variable as a function of model parameters, in order to keep the system flexible enough to easily accommodate modifications.

As a starting point for the recursive process, we make two arbitrary choices regarding three of the endogenous variables. First, we normalize the steady-state real exchange rate to $s = 1$. Thus, the real exchange rate, s, is treated as an exogenous parameter in the steady-state solution. Instead, world trade, wt^*, is treated as an endogenous variable whose steady-state value is solved during the process, and therefore we still end up with a steady-state system involving 48 equations and 48 endogenous variables. Second, we set $s_D = s_{DX} = 0$, i.e. dividends are zero in the steady state, even though the model contains monopolistic firms with positive markups. This is consistent with the assumption of fixed production costs $\left(\psi \text{ and } \psi^X\right)$ in the production technologies (17 and 46) and, at the same time, facilitates free entry while maintaining a constant number of firms. We again treat s_D and s_{DX} as if they were exogenous parameters and the fixed costs, ψ and ψ^X, as if they were endogenous variables to be solved. We therefore again end up with a steady-state system of 48 equations with 48 endogenous variables to be solved. Once we have chosen the steady-state values of the real exchange rate and dividends, it is now possible to show the recursive solution of the steady-state values of the 48 variables:

$$wt^*, \psi, \psi^X, p_Y, p_Y^M, p_X, p_I, p_G, p_{DX}, p_H, p_{IM}, mc, r_K, w, R^F, N, u, k, k^s, \lambda, Q, y, c, i, g, x,$$
$$q^C, q^I, q^G, q^X, h, h^s, h^C, h^I, h^G, h^X, im^C, im^I, im^G, im^X, s_T, s_B, s_X, s_{IM}, s_{TB}, p_{DX}, R, R^*.$$

The steady-state solution is affected by the following subset of the model's parameters:

$$s, s_D, s_{DX}, \alpha, \beta, \zeta, \delta, g_z, \kappa, \mu^C, \mu^I, \mu^G, \mu^X, \mu^*, \nu_C, \nu_I, \nu_G, \nu_X, \nu^*, \nu^F, p_X^C, \Pi, s_G,$$
$$\tau^C, \tau^D, \tau^K, \tau^N, \tau^{W_h}, \tau^{W_f}, \varphi^W, \varphi^*, \varphi^H, \varphi^X, \tilde{z}, s_{FTR}, p_{OIL}^*, \omega^*, \Delta inv.$$

The next step in the recursive solution is to adopt the full utilization assumption (116):

$$u = 1. \tag{158}$$

C.2.1 Solving for prices

Substituting the market-clearing condition (151) and the export profit definition (156) into nominal GDP (152) yields:

$$p_Y y = p_H h + p_Y y s_{DX}.$$

Using (155) we obtain:

$$p_Y = (1 - s_{DX})^{-1} p_H.$$

Substituting this into (123) make it possible to solve for the ratio of the import price to domestic price:

$$p_{IM\_H} \left(= \frac{p_{IM}}{p_H} \right) = \varphi^* \left(p_{OIL}^*\right)^{\omega^*} s \left(1 - s_{DX}\right)^{-1}. \tag{159}$$

We use the final consumption good pricing equation (132) to solve for p_H:

$$\frac{1}{p_H} = \left[\nu_C + (1-\nu_C)\left(\frac{p_{IM}}{p_H}\right)^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}},$$

and using (159) we obtain:

$$p_H = (A_C)^{-1} \quad \text{with} \quad A_C = \left[\nu_C + (1-\nu_C)\left(p_{IM\_H}\right)^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}}, \tag{160}$$

$$p_Y = (1-s_{DX})^{-1}(A_C)^{-1} \tag{161}$$

and

$$p_{IM} = \varphi^* \left(p^*_{OIL}\right)^{\omega^*} s \cdot (1-s_{DX})^{-1}(A_C)^{-1}. \tag{162}$$

From (122) we have:

$$mc = \frac{1}{\varphi^H}(A_C)^{-1}. \tag{163}$$

The final investment good pricing equation (133) is used to solve for p_I:

$$p_I = p_H\left[\nu_I + (1-\nu_I)\left(\frac{p_{IM}}{p_H}\right)^{1-\mu_I}\right]^{\frac{1}{1-\mu_I}},$$

and using (159) and (160) we obtain:

$$p_I = (A_C)^{-1} A_I \quad \text{with} \quad A_I = \left[\nu_I + (1-\nu_I)\left(p_{IM\_H}\right)^{1-\mu_I}\right]^{\frac{1}{1-\mu_I}}. \tag{164}$$

The final government consumption good pricing equation (134) is used to solve for p_G:

$$p_G = p_H\left[\nu_G + (1-\nu_G)\left(\frac{p_{IM}}{p_H}\right)^{1-\mu_G}\right]^{\frac{1}{1-\mu_G}},$$

and using (159) and (160) we obtain:

$$p_G = (A_C)^{-1} A_G \quad \text{with} \quad A_G = \left[\nu_G + (1-\nu_G)\left(p_{IM\_H}\right)^{1-\mu_G}\right]^{\frac{1}{1-\mu_G}}. \tag{165}$$

The final export good pricing equation (135) is used to solve for p_{DX} :

$$p_{DX} = p_H\left[\nu_X + (1-\nu_X)\left(\frac{p_{IM}}{p_H}\right)^{1-\mu_X}\right]^{\frac{1}{1-\mu_X}},$$

and using (159) and (160) we obtain:

$$p_{DX} = (A_C)^{-1} A_X \quad \text{with} \quad A_X = \left[\nu_X + (1-\nu_X)\left(p_{IM\_H}\right)^{1-\mu_X}\right]^{\frac{1}{1-\mu_X}}. \tag{166}$$

Using (136), we can solve for the export price (in foreign markets):

$$p_x = \frac{\varphi^X p_{DX}}{s \cdot p_Y}. \tag{167}$$

We set the value of p^*_{OIL} so that:

$$\varphi^* \left(p^*_{OIL}\right)^{\omega^*} s \left(1 - s_{DX}\right)^{-1} = 1.$$

This condition is imposed in order to avoid a direct effect of the home bias shocks (the ν's) on the prices of the final goods. In this case we obtain: $p_Y = p_I = p_G = p_H = p_{IM} = p_{DX} = 1$, $mc = 1/\varphi^H$ and $p_X = \varphi^X$. Another advantage of this calibration is that the steady-state solution is invariant to the elasticities of substitution (the μ's).

C.2.2 Solving for input prices and the trade balance

It is straightforward to solve for R, R^*, R^F and Q using (111), (112), (119) and (114):[56]

$$R = \frac{g_z \Pi}{\beta}; \tag{168}$$

$$R^* = \frac{g_z \Pi}{\beta}; \tag{169}$$

$$R^F = 1 + \nu^F (R - 1); \tag{170}$$

and

$$Q = p_I. \tag{171}$$

[56] We also assume that the domestic and foreign long-run inflation targets are identical.

From (115) we have:

$$r_K = \gamma_{u,1} p_I, \tag{172}$$

where

$$\gamma_{u,1} = \left[\frac{\beta}{g_z} - \left(1 - \delta + \tau^K \delta\right)\right] \left(1 - \tau^K\right)^{-1}.$$

We can now solve for the real wage using (121):

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\left(1 + \tau^{W_f}\right)^{1-\alpha}} mc \left(r_K\right)^{-\alpha} \left(R^F\right)^{-(1-\alpha)}\right]^{\frac{1}{1-\alpha}}. \tag{173}$$

Assuming a balanced current account in the steady state and using (144), we obtain:

$$s_{TB} = -s_{FTR}. \tag{174}$$

C.2.3 Solving for quantities per working hours

Using equation (145), $k^s = k$.

From (120):

$$\left(\frac{k}{N}\right) = \frac{\alpha}{(1-\alpha)} g_z \frac{\left(1 + \tau^{W_f}\right) R^F w}{r_K}. \tag{175}$$

Combining (113) with (175) we obtain:

$$\left(\frac{i}{N}\right) = \frac{i}{k} \left(\frac{k}{N}\right) = \left(1 - \frac{1-\delta}{g_z}\right) \left(\frac{k}{N}\right). \tag{176}$$

From (157):

$$\left(h^s + \psi\right) = \left(1 - s_D\right) \frac{p_Y y}{mc}.$$

Substituting this into the domestic intermediate goods production function (118), dividing by N and rearranging yields:

$$\left(\frac{y}{N}\right) = \left(1 - s_D\right)^{-1} \left(\left(g_z\right)^{-1} \frac{k}{N}\right)^\alpha \frac{mc}{p_Y}. \tag{177}$$

Dividing (140) by N, we obtain:

$$\left(\frac{g}{N}\right) = s_G \frac{p_Y}{p_G} \left(\frac{y}{N}\right). \tag{178}$$

C.2.4 Solving for hours worked (N)

The aggregate resource constraint (153) in per-capita terms is given by:

$$p_Y \frac{y}{N} = \frac{c}{N} + p_I \frac{i}{N} + \Delta inv \cdot p_I \frac{y}{N} + p_G \frac{g}{N} + \frac{s \cdot p_X p_Y x - p_{IM} im}{N}.$$

Rearranging and using (141), (142) and (143):

$$\left(\frac{c}{N}\right) = p_Y \left(\frac{y}{N}\right) - p_I \left(\frac{i}{N}\right) - \Delta inv \cdot p_I \left(\frac{y}{N}\right) - p_G \left(\frac{g}{N}\right) - s_{TB} p_Y \left(\frac{y}{N}\right). \tag{179}$$

Combining the equation for household wage-setting (117) with the first-order condition for consumption (110) we have:

$$\left(1 - \tau^N - \tau^{W_h}\right) w = \varphi^W N^\zeta (\lambda)^{-1} = \varphi^W N^\zeta \left(\frac{(1 - \kappa/g_z)^{-1}}{1 + w_{\tau^C} \tau^C} \frac{1}{c}\right)^{-1}.$$

Rearranging to solve for N results in:

$$N = \left[\varphi^W \left(1 + w_{\tau^C} \tau^C\right) (1 - \kappa/g_z) \left(1 - \tau^N - \tau^{W_h}\right)^{-1} \left(\frac{c}{N}\right) w^{-1}\right]^{-\left(\frac{1}{\zeta+1}\right)}. \tag{180}$$

C.2.5 Returning to levels

Having solved for N, it is straightforward to return to levels:

$$c = \left(\frac{c}{N}\right) N; \tag{181}$$

$$y = \left(\frac{y}{N}\right) N; \tag{182}$$

$$g = \left(\frac{g}{N}\right) N; \tag{183}$$

$$i = \left(\frac{i}{N}\right) N; \tag{184}$$

$$k = \left(\frac{k}{N}\right) N. \tag{185}$$

From (145), (155) and (146), we set:

$$k^s = k; \tag{186}$$

$$h^s = y; \tag{187}$$

$$h = h^s. \tag{188}$$

C.2.6 Solving for sector-specific intermediate goods, imports and exports

Based on market clearing in the C, I and G final goods markets (148, 149 and 150):

$$q^C = c; \tag{189}$$

$$q^I = i + \Delta inv \cdot y; \tag{190}$$

and

$$q^G = g. \tag{191}$$

Using the demand for domestic and intermediate goods in the C, I and G final goods markets (124-126 and 128-130):

$$h^C = \nu_C \left(p_H\right)^{-\mu_C} q^C; \tag{192}$$

$$h^I = \nu_I \left(\frac{p_H}{p_I}\right)^{-\mu_I} q^I; \tag{193}$$

$$h^G = \nu_G \left(\frac{p_H}{p_G}\right)^{-\mu_G} q^G; \tag{194}$$

$$im^C = (1 - \nu_C) \left(p_{IM}\right)^{-\mu_C} q^C; \tag{195}$$

$$im^I = (1 - \nu_I) \left(\frac{p_{IM}}{p_I}\right)^{-\mu_I} q^I; \tag{196}$$

and

$$im^G = (1 - \nu_G) \left(\frac{p_{IM}}{p_G}\right)^{-\mu_G} q^G. \tag{197}$$

We can now solve for h_X using the relationship $h = h_C + h_I + h_G + h_X$ (which is not one of our 48 steady-state equations, but can be shown to hold in steady state):

$$h^X = h - h^C - h^I - h^G. \tag{198}$$

We can now use (127) to solve for q^X:

$$q^X = (\nu_X)^{-1} \left(\frac{p_H}{p_{DX}} \right)^{\mu_X} h^X, \tag{199}$$

and (131) to solve for im_X :

$$im^X = (1 - \nu_X) \left(\frac{p_{IM}}{p_{DX}} \right)^{-\mu_G} q^X. \tag{200}$$

We can solve for total import using the market-clearing condition (147):

$$im = im^C + im^I + im^G + im^X. \tag{201}$$

The import share, export share and export level are calculated from (143), (141) and (142):

$$s_{IM} = \frac{p_{IM} im}{p_Y y}; \tag{202}$$

$$s_X = s_{TB} + s_{IM}; \tag{203}$$

and

$$x = \frac{s_X y}{s \cdot p_X}. \tag{204}$$

World trade, wt^*, is derived from the inverse of the export demand equation (137):

$$wt^* = (\nu^* \tilde{z})^{-1} (p_X^{c,*})^{-\mu^*} (p_X)^{\mu^*} x. \tag{205}$$

C.2.7 Solving for government debt

From (139):

$$s_T = 0. \tag{206}$$

Rearranging the government budget constraint (138) to solve for s_B yields:

$$\begin{aligned} s_B \;=\; & \left[w_{\tau^C} \tau^C \frac{c}{p_Y y} + \left(\tau^N + \tau^{W_h} + \tau^{W_f}\right) \frac{wN}{p_Y y} \right] \cdot \left[(\Pi g_z)^{-1} - R^{-1} \right]^{-1} \\ & + \left[\frac{\tau^K}{g_z} \left(r_K - \delta p_I\right) \frac{k}{p_Y y} + \tau^D s_D + s_T - s_G \right] \cdot \left[(\Pi g_z)^{-1} - R^{-1} \right]^{-1} . \end{aligned} \tag{207}$$

C.2.8 Solving for fixed costs

Combining (157) with (155):

$$\psi = \left[(1 - s_D) \frac{p_Y}{mc} - 1 \right] y \tag{208}$$

and from (156):

$$\psi^X = \left(\frac{s \cdot p_X p_Y}{p_{DX}} - 1 \right) x - s_{DX} \frac{p_Y y}{p_{DX}}. \tag{209}$$

C.2.9 Solving for additional variables

Equation (110) is now the solution to the consumer's marginal utility from income:

$$\lambda = \frac{(1 - \kappa / g_z)^{-1}}{1 + w_{\tau^C} \tau^C} \frac{1}{c}. \tag{210}$$

From (154), we can solve for the output deflator (in market prices):

$$p_Y^M = \left(1 + w_{\tau^C} \tau^C\right) \frac{c}{y} + p_I \frac{i}{y} + \Delta inv \cdot p_I + p_G \frac{g}{y} + s_{TB} p_Y. \tag{211}$$

Appendix D The log-linearized model

This appendix presents the log-linearized system of equations. In what follows, FOC stands for "First Order Condition", and WRT stands for "With Respect To".

D.1 Households' Choice of Allocations

Marginal utility from consumption:

$$\hat{\lambda}_t = \left(- \left(\frac{1}{1 - \kappa\, g_z^{(-1)}} \right) \right) \hat{c}_t + \frac{\kappa\, g_z^{(-1)}}{1 - \kappa\, g_z^{(-1)}} \hat{c}_{t-1} - \frac{\kappa\, g_z^{(-1)}}{1 - \kappa\, g_z^{(-1)}} \hat{g_{z,t}} - \frac{1}{1 + 0.78\, \tau^c} \hat{\tau}_t^C + \hat{\epsilon}_t^C . \tag{212}$$

FOC WRT K_{t+1} (Tobin's Q):

$$\begin{aligned}\hat{Q}_t &= \frac{\beta\,(1-\delta)}{g_z} E_t(\hat{Q}_{t+1}) + E_t(\hat{\lambda}_{t+1}) - \hat{\lambda}_t - E_t(\hat{g_{z,t+1}}) \\ &-\frac{\beta\,\left(1-\tau^K\right)\,\gamma_{u,1}}{g_z} E_t\,\left(\frac{1}{1-\tau^K}\,\hat{\tau}^K_{t+1} - \hat{r_{k,t+1}}\right) + \frac{\beta\,\delta}{g_z} E_t\,\left(\hat{\tau}^K_{t+1} + \tau^K\,\hat{p_{I,t+1}}\right).\end{aligned} \tag{213}$$

FOC WRT Investment:

$$\begin{aligned}\hat{\imath}_t &= \frac{1}{1+(1-\omega_{\Gamma_I})^2\,\beta + \omega^2_{\Gamma_I}\,\beta^2}\left(\begin{array}{c}\omega_{\Gamma_I}\,\beta^2\,E_t(\hat{\imath}_{t+2}) + (1-\omega_{\Gamma_I})\,\beta\,(1-\omega_{\Gamma_I}\,\beta)\,E_t(\hat{\imath}_{t+1}) \\ +(1-\omega_{\Gamma_I})\,(1-\omega_{\Gamma_I}\,\beta)\,\hat{\imath}_{t-1} + \omega_{\Gamma_I}\,\hat{\imath}_{t-2}\end{array}\right) \\ &+\frac{1}{1+(1-\omega_{\Gamma_I})^2\,\beta + \omega^2_{\Gamma_I}\,\beta^2}\left(\begin{array}{c}\omega_{\Gamma_I}\,\beta^2\,E_t(\hat{g_{z,t+2}}) + \left(\omega^2_{\Gamma_I}\,\beta^2 + (1-\omega_{\Gamma_I})\,\beta\right)E_t(\hat{g_{z,t+1}}) \\ +\hat{g_{z,t}}\,\left(\beta\,\omega_{\Gamma_I}\,(1-\omega_{\Gamma_I}) - 1\right) + \omega_{\Gamma_I}\,\hat{g_{z,t-1}}\end{array}\right) \\ &+\frac{1}{\left(1+(1-\omega_{\Gamma_I})^2\,\beta + \omega^2_{\Gamma_I}\,\beta^2\right)\,\gamma_I\,g_z^{2\,(1-\omega_{\Gamma_I})}}\,\left(\hat{Q}_t - \hat{p_{I,t}} + \hat{\epsilon}^I_t\right).\end{aligned} \tag{214}$$

FOC WRT Capital Utilization:

$$\hat{r}_{k,t} = \hat{p}_{I,t} + \frac{\gamma_{u,2}}{\gamma_{u,1}}\,\hat{u}_t. \tag{215}$$

FOC WRT Domestic Bonds:

$$\hat{\lambda}_{t+1} - \hat{\lambda}_t - g_{z,t+1} + \hat{r}_t - \hat{\pi}_{C,t+1} + \hat{\epsilon}^{RP}_t + \hat{\epsilon}^{DRP}_t = 0. \tag{216}$$

Modified (Risk-adjusted) UIP Condition:

$$\begin{aligned}\hat{r}_t - \hat{r}^*_t &= (1-\gamma_S)\,(E_t\,\left(\hat{s}_{t+1}\right) - \hat{s}_t) + E_t(\hat{\pi}_{Y,t+1} - \hat{\pi}^*_{Y,t+1}) - \gamma_S\,\left(\hat{s}_t - \hat{s}_{t-1} + \hat{\pi}_{Y,t} - \hat{\pi}^*_{y,t}\right) \\ &+\gamma_S\,(E_t\,(\hat{\bar{\pi}}_{t+1} - \hat{\bar{\pi}}^*_{t+1}) + (\hat{\bar{\pi}}_t - \hat{\bar{\pi}}^*_t)) - \gamma_B\,E_t(\hat{s}_{B^*,t+1}) + \hat{\epsilon}^{RP^*}_t - \hat{\epsilon}^{DRP}_t.\end{aligned} \tag{217}$$

Capital Accumulation:

$$\hat{k}_{t+1} = g_z^{(-1)}\,(1-\delta)\,\hat{k}_t - \hat{g}_{z,t}\,g_z^{(-1)}\,(1-\delta) + \hat{\epsilon}^I_t\,\left(1 - g_z^{(-1)}\,(1-\delta)\right) + \hat{\imath}_t\,\left(1 - g_z^{(-1)}\,(1-\delta)\right). \tag{218}$$

D.2 Labor Supply and Wage Setting

After-tax real wage:

$$\hat{w}_{\tau,t} = \hat{w}_t - \left(\frac{1}{1-\tau^N-\tau^{W_h}}\,\hat{\tau}^N_t + \frac{1}{1-\tau^N-\tau^{W_h}}\,\hat{\tau}^{W_h}_t\right). \tag{219}$$

MRS between consumption and leisure:

$$\widehat{mrs}_t = \hat{\epsilon}_{N,t} + \zeta\, \hat{n}_t - \hat{\lambda}_t. \tag{220}$$

Nominal-wage Inflation:

$$\begin{aligned} & \hat{g}_{z,t} + \hat{w}_t - \hat{w}_{t-1} + \hat{\pi_{C,t}} - \left(\chi_W\, \hat{\pi}_{C,t-1} + \hat{\bar{\pi}}_t\, (1-\chi_W) + \hat{g}_{z,t-1}\, \chi_{W,g_z}\right) \\ = & \;\beta\, \left(E_t(\hat{g}_{z,t+1}) + E_t(\hat{\pi}_{C,t+1}) + E_t(\hat{w}_{t+1}) - \hat{w}_t - \left(\hat{\pi}_{C,t}\, \chi_W + E_t(\hat{\bar{\pi}}_{t+1}\, (1-\chi_W)) + \hat{g}_{z,t}\, \chi_{W,g_z}\right)\right) \\ & - \frac{(1-\beta\,\xi_W)\,(1-\xi_W)}{\xi_W\, \left(1+\zeta\, \frac{\phi_w}{\phi_w-1}\right)} \left(\begin{array}{c} \hat{w}_t - \left(\frac{1}{1-\tau^N-\tau^{W_h}}\, \hat{\tau}_t^N + \frac{1}{1-\tau^N-\tau^{W_h}}\, \hat{\tau}_t^{W_h}\right) \\ -\left(\hat{\epsilon}_{N,t} + \zeta\, \hat{n}_t - \hat{\lambda}_t\right) - \hat{\phi}_{w,t} \end{array} \right). \end{aligned} \tag{221}$$

D.3 Intermediate-good Firms Resource Allocation

Production technology:

$$\hat{h}_{s,t} = \left(1 + \psi\, h^{(-1)}\right)\, \left(\hat{\epsilon}_t + \alpha\, \left(\hat{k^s}_t - \hat{g}_{z,t}\right) + \hat{n}_t\, (1-\alpha)\right). \tag{222}$$

Resource allocation:

$$\hat{r}_{k,t} = \hat{w}_t + \hat{n}_t + \left(1+\tau^{W_f}\right)^{(-1)}\, \hat{\tau}_t^{W_f} + \hat{R_{f,t}} - \left(\hat{k}_t^s - \hat{g}_{z,t}\right). \tag{223}$$

Real marginal cost (in terms of CPI):

$$\widehat{mc}_t = (-\hat{\epsilon}_t) + \hat{r}_{k,t}\, \alpha + (1-\alpha)\, \left(\hat{R}_{f,t} + \hat{w}_t + \left(1+\tau^{W_f}\right)^{(-1)}\, \hat{\tau}_t^{W_f}\right). \tag{224}$$

Interest rate on working capital:

$$\hat{R}_{f,t} = \frac{\nu^f\, R}{1-\nu^f\, (R-1)}\, \left(\hat{r}_t + \hat{\epsilon}_t^{RP} + \hat{\epsilon}_t^F\right) + \frac{\nu^f\, (R-1)}{1-\nu^f\, (R-1)} \hat{\nu}_{f,t}. \tag{225}$$

D.4 Intermediate-good Firms Price Setting (Phillips Curves)

Phillips Curve of the domestic intermediate goods firm:

$$\begin{aligned} \hat{\pi}_{H,t} - \hat{\bar{\pi}}_t \;=\; & \frac{\beta}{1+\beta\,\chi_H}\, \left(E_t(\hat{\pi}_{H,t+1}) - E_t(\hat{\bar{\pi}}_{t+1})\right) + \frac{\chi_H}{1+\beta\,\chi_H}\, \left(\hat{\pi}_{H,t-1} - \hat{\bar{\pi}}_t\right) \\ & + \frac{\beta\,\chi_H}{1+\beta\,\chi_H}\, \left(E_t(\hat{\bar{\pi}}_{t+1}) - \hat{\bar{\pi}}_t\right) + \frac{(1-\beta\,\xi_H)\,(1-\xi_H)}{(1+\beta\,\chi_H)\,\xi_H}\, \left(\widehat{mc}_t^H + \hat{\varphi}_t^H\right). \end{aligned} \tag{226}$$

Real marginal cost (in terms of domestic price index):

$$\widehat{mc}_t^H = \widehat{mc}_t - \hat{p}_{H,t}. \tag{227}$$

Real domestic price index (in terms of CPI):

$$\hat{p}_{H,t} = \hat{\pi}_{H,t} + \hat{p}_{H,t-1} - \hat{\pi}_{C,t}. \tag{228}$$

Phillips Curve of the foreign intermediate goods firm: (denominated in terms of the local currency):

$$\begin{aligned} \hat{\pi}_{IM,t} - \hat{\bar{\pi}}_t \quad = \quad & \frac{\beta^*}{1+\beta^*\,\chi^*}\,E_t(\hat{\pi}_{IM,t+1} - \hat{\bar{\pi}}_{t+1} + \frac{\chi^*}{1+\beta^*\,\chi^*}\,\left(\hat{\pi}_{IM,t-1} - \hat{\bar{\pi}}_t\right) \\ & +(E_t\left(\hat{\bar{\pi}}_{t+1}\right) - \hat{\bar{\pi}}_t)\,\frac{\beta^*\,\chi^*}{1+\beta^*\,\chi^*} + \frac{(1-\beta^*\,\xi^*)\,(1-\xi^*)}{(1+\beta^*\,\chi^*)\,\xi^*}\,(\widehat{mc}_{IM,t} + \hat{\varphi}_t^*)\,. \end{aligned} \tag{229}$$

Real marginal cost of the foreign intermediate goods firm in terms of the price of imported goods:

$$\widehat{mc}_{IM,t} = \hat{s}_t + \hat{p}_{Y,t} + \omega^*\,\left(\hat{p}_{OIL,t} - \left(\hat{\pi}^*_{y,t} + \hat{p}_{OIL,t} - \hat{p}_{OIL,t-1}\right)\right) - \hat{p}_{IM,t}. \tag{230}$$

Real price of imported goods (local currency, in terms of CPI):

$$\hat{p}_{IM,t} = \hat{\pi}_{IM,t} + \hat{p}_{IM,t-1} - \hat{\pi}_{C,t}. \tag{231}$$

D.5 Final-Good Firms: Technology, Inputs and Prices

In what follows, prices are expressed in real terms, with the denominator being the CPI. e.g., $\hat{p}_{H,t} \equiv \ln\left(P_{H,t}/P_{C,t}\right)$.

D.5.1 Final private consumption good

Demand for domestic intermediate goods:

$$\hat{h}_{C,t} = \hat{q}_{C,t} - \hat{p}_{H,t}\,\mu_C + \hat{\nu}_{C,t}. \tag{232}$$

Demand for imported intermediate goods:

$$\widehat{im}_{C,t} = \hat{q}_{C,t} - \mu_c \left(\hat{p}_{IM,t} - \hat{\Gamma}^{+}_{IM^{C},t}\right) - \hat{\nu}_{C,t}\,\frac{\nu_E}{1-\nu_E}. \tag{233}$$

Import share adjustment costs:

$$\hat{\Gamma}^{+}_{IM^{C},t} = \left(-\gamma^{C}_{IM}\right)\left(\widehat{im}_{C,t} - \hat{q}_{C,t} - \left(\widehat{im}_{C,t-1} - \hat{q}_{C,t-1}\right)\right) + \hat{\epsilon}^{IM}_{t}. \tag{234}$$

Price of the private consumption good:

$$0 = \hat{p}_{H,t}\,\nu_C\,p_H^{1-\mu_C} + \left(\hat{p}_{IM,t} - \hat{\Gamma}^{+}_{IM^{C},t}\right)(1-\nu_C)\,p_{IM}^{1-\mu_C} + \hat{\nu}_{ct}\,\frac{\nu_C}{1-\mu_C}\left(p_H^{1-\mu_C} - p_{IM}^{1-\mu_C}\right). \tag{235}$$

D.5.2 Final investment good

Demand for domestic intermediate goods:

$$\hat{h}_{I,t} = \hat{q}_{I,t} - \mu_I\left(\hat{p}_{H,t} - \hat{p}_{I,t}\right) + \hat{\nu}_{I,t}. \tag{236}$$

Demand for imported intermediate goods:

$$\widehat{im}_{I,t} = \hat{q}_{I,t} - \mu_I\left(\hat{p}_{IM,t} - \hat{p}_{I,t} - \hat{\Gamma}^{+}_{IM^{I}t}\right) - \hat{\nu}_{I,t}\,\frac{\nu_I}{1-\nu_I}. \tag{237}$$

Import share adjustment costs:

$$\hat{\Gamma}^{+}_{IM^{I},t} = \hat{\epsilon}^{IM}_{t} + \left(-\gamma^{I}_{IM}\right)\left(\widehat{im}_{I,t} - \hat{q}_{I,t} - \left(\widehat{im}_{I,t-1} - \hat{q}_{I,t-1}\right)\right). \tag{238}$$

Price of the investment good:

$$\begin{aligned}\hat{p}_{I,t} \quad = \quad & \hat{p}_{H,t}\,\nu_I\left(\frac{p_H}{p_I}\right)^{1-\mu_I} + (1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}\left(\hat{p}_{IM,t} - \hat{\Gamma}^{+}_{IM^{I},t}\right) \\ & + \hat{\nu}_{I,t}\,\frac{\nu_I}{1-\mu_I}\left(\left(\frac{p_H}{p_I}\right)^{1-\mu_I} - \left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}\right).\end{aligned} \tag{239}$$

D.5.3 Final public consumption good

Demand for domestic intermediate goods:

$$\hat{h}_{G,t} = \hat{q}_{G,t} - \mu_G\ (\hat{p}_{H,t} - \hat{p}_{G,t}) + \hat{\nu}_{G,t}. \tag{240}$$

Demand for imported intermediate goods:

$$\widehat{im}_{G,t} = \hat{q}_{G,t} - \mu_G\ \left(\hat{p}_{IM,t} - \hat{p}_{G,t} - \hat{\Gamma}^{+}_{IM^{G},t}\right) - \hat{\nu}_{G,t}\,\frac{\nu_G}{1-\nu_G}. \tag{241}$$

Import share adjustment costs:

$$\hat{\Gamma}^{+}_{IM^{G},t} = \hat{\epsilon}^{IM}_{t} + \left(-\gamma^{G}_{IM}\right)\ \left(\widehat{im}_{G,t} - \hat{q}_{G,t} - \left(\widehat{im}_{G,t-1} - \hat{q}_{G,t-1}\right)\right). \tag{242}$$

Price of the public consumption goods:

$$\begin{aligned} \hat{p}_{G,t} \quad = \quad & \hat{p}_{H,t}\,\nu_G\left(\frac{p_H}{p_G}\right)^{1-\mu_G} + (1-\nu_G)\ \left(\frac{p_{IM}}{p_G}\right)^{1-\mu_G}\left(\hat{p}_{IM,t} - \hat{\Gamma}^{+}_{IM^{G},t}\right). \\ & +\hat{\nu}_{G,t}\,\frac{\nu_G}{1-\mu_G}\ \left(\left(\frac{p_H}{p_G}\right)^{1-\mu_G} - \left(\frac{p_{IM}}{p_G}\right)^{1-\mu_G}\right). \end{aligned} \tag{243}$$

D.5.4 Final exports good

Demand for domestic intermediate goods:

$$\hat{h}_{X,t} = \hat{q}_{X,t} - \mu_X\ (\hat{p}_{H,t} - \hat{p}_{DX,t}) + \hat{\nu}_{X,t}. \tag{244}$$

Demand for imported intermediate goods:

$$\widehat{im}_{X,t} = \hat{q}_{X,t} - \mu_X\ \left(\hat{p}_{IM,t} - \hat{p}_{DX,t} - \hat{\Gamma}^{+}_{IM^{X},t}\right) - \hat{\nu}_{X,t}\,\frac{\nu_X}{1-\nu_X}. \tag{245}$$

Import share adjustment costs:

$$\hat{\Gamma}^{+}_{IM^{X},t} = \hat{\epsilon}^{X}_{t} + \left(-\gamma^{X}_{IM}\right)\ \left(\widehat{im}_{X,t} - \hat{q}_{X,t} - \left(\widehat{im}_{X,t-1} - \hat{q}_{X,t-1}\right)\right). \tag{246}$$

(Domestic) Price of the exported good:

$$\hat{p}_{DX,t} = \hat{p}_{H,t}\,\nu_x\left(\frac{p_H}{p_{DX}}\right)^{1-\mu_x} + (1-\nu_X)\left(\frac{p_{IM}}{p_{DX}}\right)^{1-\mu_X}\left(\hat{p}_{IM,t} - \hat{\Gamma}^{+}_{IM^X,t}\right)$$
$$+\hat{\nu}_{X,t}\frac{\nu_X}{1-\mu_X}\left(\left(\frac{p_H}{p_{DX}}\right)^{1-\mu_X} - \left(\frac{p_{IM}}{p_{DX}}\right)^{1-\mu_X}\right). \tag{247}$$

D.5.5 Exports to foreign markets (monopolistic firms)

Phillips Curve for the price of the exported good (in foreign currency):

$$\hat{\pi}_{X,t} - \hat{\bar{\pi}}^{*}_{t} = \frac{\beta}{1+\beta\,\chi_X}E_t\left(\hat{\pi}_{X,t+1} - \hat{\bar{\pi}}^{*}_{t+1}\right) + \frac{\chi_X}{1+\beta\,\chi_X}\left(E_t(\hat{\bar{\pi}}^{*}_{t+1}) - \hat{\bar{\pi}}^{*}_{t}\right)$$
$$+\frac{\beta\,\chi_X}{1+\beta\,\chi_X}\left(E_t(\hat{\bar{\pi}}^{*}_{t+1}) - \hat{\bar{\pi}}^{*}_{t}\right) + \frac{(1-\beta\,\xi_X)\,(1-\xi_X)}{(1+\beta\,\chi_X)\,\xi_X}\left(\widehat{mc}^{X}_{t} + \hat{\varphi}^{X}_{t}\right). \tag{248}$$

Exporters' real marginal cost (WRT the price of the exported good):

$$\widehat{mc}^{X}_{t} = \hat{p}_{DX,t} - \hat{s}_t - \hat{p}_{X,t} - \hat{p}_{Y,t}. \tag{249}$$

Real Price of the exported good (foreign currency, WRT foreign output deflator):

$$\hat{p}_{X,t} = \hat{\pi}_{X,t} + \hat{p}_{X,t-1} - \hat{\pi}^{*}_{Y,t}. \tag{250}$$

D.6 The Fiscal and Monetary Authorities

Government consumption:

$$\hat{g}_t = \rho_G\,\hat{g}_{t-1} + \eta^{G}_{t}. \tag{251}$$

Direct consumption tax-rate:

$$\hat{\tau}^{C}_{t} = \rho_{\tau^C}\,\hat{\tau}^{C}_{t-1} + \eta^{\tau^C}_{t}. \tag{252}$$

Dividend tax:

$$\hat{\tau}^{D}_{t} = \rho_{\tau^D}\,\hat{\tau}^{D}_{t-1} + \eta^{\tau^D}_{t}. \tag{253}$$

Capital rental tax:

$$\hat{\tau}_t^k = \rho_{\tau^K} \, \hat{\tau}_{t-1}^K + \eta_t^{\tau^K}. \tag{254}$$

Income tax on households:

$$\hat{\tau}_t^N = \rho_{\tau^N} \, \hat{\tau}_{t-1}^N + \eta_t^{\tau^N}. \tag{255}$$

Additional pay-roll tax on households:

$$\hat{\tau}_t^{W_h} = \rho_{\tau^{W_h}} \, \hat{\tau}_{t-1}^{W_h} + \eta_t^{\tau^{W_h}}. \tag{256}$$

Additional pay-roll tax on firms:

$$\hat{\tau}_t^{W_f} = \rho_{\tau^{W_f}} \, \hat{\tau}_{t-1}^{W_f} + \eta_t^{\tau^{W_f}}. \tag{257}$$

Output growth:

$$\hat{g}_{Y,t} = \hat{g}_{z,t} + \hat{y}_t - \hat{y}_{t-1}. \tag{258}$$

Taylor Rule:

$$\hat{r}_t = \phi_R \, \hat{r}_{t-1} + (1 - \phi_R) \left(\hat{\bar{\pi}}_t + \hat{rr}_t^{fw} + \phi_\Pi \left(\hat{\pi}_t^{CB} - \bar{\pi}_t \right) + \phi_y \, y_t^{GAP} + \phi_{\Delta S} \, \Delta \hat{S}_t \right) + \eta_t^R. \tag{259}$$

Time-varying inflation objective:

$$\hat{\bar{\pi}}_t = \rho_{\bar{\pi}} \, \hat{\bar{\pi}}_{t-1} + \eta_t^{\overline{\Pi}}. \tag{260}$$

Real interest rate:

$$\hat{ri}_t = \hat{r}_t - \hat{\pi}_{C,t+1}. \tag{261}$$

Forward 5-10 years expected real interest rate:

$$\hat{rr}_t^{fw} = \frac{1}{20} E_t \left(\hat{ri}_{t+20} + ... + \hat{ri}_{t+39} \right). \tag{262}$$

Nominal Depreciation:

$$\Delta \hat{S}_t = \hat{s}_t - \hat{s}_{t-1} + \hat{\pi}_{Y,t} - \hat{\pi}_{Y,t}^*. \tag{263}$$

Deviation of production inputs from trend:

$$y_t^{GAP} = \hat{y}_t - \hat{\epsilon}_t. \tag{264}$$

The inflation that the CB reacts to:

$$\hat{\pi}_t^{CB} = 0.25\ \left(E_t(\hat{\pi}_{C,t+1}) + \hat{\pi}_{C,t} + \hat{\pi}_{C,t-1} + \hat{\pi}_{C,t-2}\right). \tag{265}$$

D.7 Net Foreign Assets and the Current Account

Ratio of trade balance to domestic output:

$$\hat{s}_{TB,t} = \hat{s}_{X,t} - \hat{s}_{IM,t}. \tag{266}$$

Ratio of exports to domestic output:

$$\hat{s}_{X,t} = \frac{s\, p_X\, x}{y}\ \left(\hat{s}_t + \hat{p}_{X,t} + \hat{x}_t - \hat{y}_t\right). \tag{267}$$

Ratio of imports to domestic output:

$$\hat{s}_{IM,t} = s_{IM}\ \left(\hat{p}_{IM,t} + \widehat{im}_t - \hat{p}_{Y,t} - \hat{y}_t\right). \tag{268}$$

Ratio of current account to domestic output:

$$(CA/Y)_t = \hat{s}_{TB,t} + \hat{s}_{FTR,t}. \tag{269}$$

Ratio of "foreign transfers" to domestic output:

$$\hat{s}_{FTR,t} = \rho_{FTR}\, \hat{s}_{FTR,t-1} + \eta_t^{FTR}. \tag{270}$$

Export demand:

$$\hat{x}_t = \hat{\hat{z}}_t + \widehat{im}_t^* + \hat{\nu}_t^* - \mu^*\ \left(\hat{p}_{x,t} - \hat{p}_{x,t}^* - (-\gamma^*)\ \left(\hat{x}_t - \widehat{im}_t^* - \hat{\hat{z}}_t - \left(\hat{x}_{t-1} - \widehat{im}_{t-1}^* - \hat{\hat{z}}_{t-1}\right)\right)\right). \tag{271}$$

Ratio of net foreign assets to domestic output:

$$E_t(s_{B_{t+1}^*})\, r^{*(-1)} = (CA/Y)_t + g_z^{(-1)}\, \pi^{*(-1)}\, E_{t-1}(s_{B_t^*}) s_{B^*}. \tag{272}$$

Real exchange rate in CPI terms:

$$\hat{s}_t = \hat{c}_t + \hat{p}_{Y,t}. \tag{273}$$

D.8 Market Clearing Conditions

Market clearing in the domestic intermediate good markets:

$$\hat{h}_t^s = \hat{h}_t. \tag{274}$$

Market clearing in the final consumption good market:

$$\hat{q}_{C,t} = \hat{c}_t. \tag{275}$$

Market clearing in the final investment good market:

$$\hat{q}_{I,t} = \hat{\imath}_t \, \frac{i}{q_i} + \hat{u}_t \, \frac{\gamma_{u,1}}{g_z} \, \frac{k}{q_i} + \frac{y}{q_i} \, \left(\Delta\hat{inv}_t + \hat{y}_t \, \Delta inv \right). \tag{276}$$

Market clearing in the final government good market:

$$\hat{q}_{G,t} = \hat{g}_t. \tag{277}$$

Market clearing in the final export good market:

$$\hat{q}_{X,t} = \hat{x}_t \, \frac{x}{q_x}. \tag{278}$$

Market clearing in the capital market:

$$\hat{u}_t + \hat{k}_t = \hat{k}_{s,t}. \tag{279}$$

Aggregate resource constraint:

$$\begin{aligned} \hat{p}_{Y,t} + \hat{y}_t \quad = \quad & \hat{c}_t \, \frac{p_C c}{p_y y} + \frac{p_I \, i}{P_y y} \, \left(\hat{\imath}_t + \hat{p_{I,t}} \right) + \frac{p_I k}{p_y y} \hat{u}_t \, \gamma_{u,1} \, + \frac{p_I}{P_Y} \, \left(\hat{y}_t \, \Delta inv + \Delta\hat{inv}_t + \hat{p}_{I,t} \, \Delta inv \right) \\ & + \frac{p_G g}{p_Y g} \, (\hat{p}_{g,t} + \hat{g}_t) + \frac{s \, p_X \, x}{y} \, (\hat{x}_t + \hat{s}_t + \hat{p}_{Y,t} + \hat{p}_{X,t}) \\ & - \frac{p_{IM} \, im_C}{p_Y y} \, \left(\hat{p}_{IM,t} + \widehat{im}_{C,t} - \hat{\Gamma}^+_{IM^C,t} \right) - \frac{p_{IM} \, im_I}{p_Y y} \, \left(\hat{p}_{IM,t} + \widehat{im}_{I,t} - \hat{\Gamma}^+_{IM^I,t} \right) \\ & - \frac{p_{IM} \, im_G}{p_Y y} \, \left(\hat{p}_{IM,t} + \widehat{im}_{G,t} - \hat{\Gamma}^+_{IM^G,t} \right) - \frac{p_{IM} \, im_X}{p_Y y} \, \left(\hat{p}_{IM,t} + \widehat{im}_{X,t} - \hat{\Gamma}^+_{IM^X,t} \right). \end{aligned} \tag{280}$$

Inflation of GDP deflator:

$$\hat{\pi}_{Y,t} = \hat{\pi}_{C,t} + \hat{p}_{y,t} - \hat{p}_{y,t-1}. \tag{281}$$

Output deflator:

$$\hat{p}_{Y,t} + \hat{y}_t = \frac{h\,p_H}{y\,p_Y}\left(\hat{p}_{H,t} + \hat{h}_t\right) + \frac{x\,s\,p_X\,p_Y}{P_Y Y}\left(\hat{c}_t + \hat{p}_{X,t} + \hat{p}_{Y,t} + \hat{x}_t\right) - \frac{p_{DX}\,q_X}{P_Y Y}\left(\hat{q}_{X,t} + \hat{p}_{DX,t}\right). \tag{282}$$

Total imported intermediate goods:

$$\hat{im}_t = \frac{im_C}{im}\widehat{im}_{C,t} + \frac{im_I}{im}\widehat{im}_{I,t} + \frac{im_G}{im}\widehat{im}_{G,t} + \frac{im_X}{im}\widehat{im}_{X,t}. \tag{283}$$

Aggregate resource constraint in market prices:

$$\begin{aligned}
\hat{y}_t + \hat{p}^M_{Y,t} \quad = \quad & \frac{(1+0.78\,\tau_c)\,c}{p^M_Y\,y}\left(\hat{c}_t + \frac{1}{1+0.78\,\tau_c}\hat{\tau}^c_t\right) + \frac{p_I i}{p^M_Y y}\left(\hat{\imath}_t + \hat{p_{I,t}}\right) + \frac{p_I k}{p^M_Y y}\hat{u}_t\,\gamma_{u,1} \\
& + \frac{p_I}{p^M_Y}\left(\hat{y}_t\,\Delta inv + \Delta\widehat{inv}_t + \hat{p}_{I,t}\,\Delta inv\right) + \frac{p_G g}{p^M_Y y}\left(\hat{p}_{G,t} + \hat{g}_t\right) \\
& + \frac{xs\,p_x\,p_y}{p^M_Y y}\left(\hat{x}_t + \hat{c}_t + \hat{p}_{Y,t} + \hat{p}_{X,t}\right) \\
& - \frac{p_{IM}\,im_C}{p^M_Y\,y}\left(\hat{p}_{IM,t} + \widehat{im}_{C,t} - \hat{\Gamma}^+_{IM^C,t}\right) - \frac{P_{IM}\,im_I}{p^M_Y\,y}\left(\hat{p}_{IM,t} + \widehat{im}_{I,t} - \hat{\Gamma}^+_{IM^I,t}\right) \\
& - \frac{p_{IM}\,im_G}{y\,p^M_Y}\left(\hat{p}_{IM,t} + \widehat{im}_{G,t} - \hat{\Gamma}^+_{IM^G,t}\right) - \frac{P_{IM}\,im_X}{p^M_Y\,y}\left(\hat{p}_{IM,t} + \widehat{im}_{X,t} - \hat{\Gamma}^+_{IM^X,t}\right).
\end{aligned} \tag{284}$$

Real GDP:

$$\hat{y}_t = \hat{h}_{s,t}. \tag{285}$$

D.9 The Foreign Economy:

Foreign Output:

$$\hat{y}^*_t = c_{y^*,+}\,E_t(\hat{y}^*_{t+1}) + (1 - c_{y^*,+})\,\hat{y}^*_{t-1} - 4c_{y^*,r}\left(\widehat{ri}^*_t - \widehat{fw}_{ri^*,t}\right) + \hat{\epsilon}_{Y^*,t}. \tag{286}$$

Foreign nominal interest rate:

$$\begin{aligned}
4\hat{r}^*_t \quad = & \\
& (1 - c_{r^*,-})\left(\begin{array}{c} 4\left(\bar{\pi}^*_t + \widehat{fw}_{ri^*,t}\right) \\ +4\,c_{r^*,\pi}\,\left(0.2\,E_t\left(\hat{\pi}^*_{y,t+1} + \hat{\pi}^*_{y,t} + \hat{\pi}^*_{y,t-1} + \hat{\pi}^*_{y,t+2} + \hat{\pi}^*_{y,t+3}\right) - \bar{\pi}^*_t\right) + \hat{y}^*_t\,c_{r^*,y} \end{array}\right) \\
& + 4c_{r^*,-}\,\hat{r}^*_{t-1} + \hat{\epsilon}_{R^*,t}.
\end{aligned} \tag{287}$$

Foreign CPI inflation:

$$\begin{aligned} 4\hat{\pi}^*_{y,t} &= 4c_{\pi^*,+}E_t(\hat{\pi}^*_{Y,t+1}) + 4\hat{\pi}^*_{Y,t-1}\,(1-c_{\pi^*,+}) + c_{\pi^*,y}\,0.5\,\left(\hat{y}^*_t + \hat{y}^*_{t-1}\right) \\ &\quad + c_{\pi^*,OIL}\hat{p}_{OIL,t} + c_{\pi^*,\Delta OIL}\,(\hat{p}_{OIL,t} - \hat{p}_{OIL,t-2}) + \hat{\pi}^*_{Y,t}. \end{aligned} \tag{288}$$

World import gap:

$$\widehat{im}^*_t = c_{wt,y}\hat{y}^*_t + c_{wt,y-}\hat{y}^*_{t-1} + c^*_{wt,-}\,\widehat{im}^*_{t-1} + \hat{\epsilon}_{IM^*_t}. \tag{289}$$

Relative foreign price of oil:

$$\hat{p}_{OILl,t} = c_{OIL,-}\hat{p}_{OIL,t-1} - c_{OIL,\Delta}\,(\hat{p}_{OIL,t-1} - \hat{p}_{OIL,t-2}) + \eta^{OIL}_t. \tag{290}$$

D.10 Observation Equations

Real per-capita GDP growth rate:

$$\Delta Y_t = \hat{g}_{z,t} + \hat{y}_t - \hat{y}_{t-1} + g_z - 1 + EX_{\Delta N,t}. \tag{291}$$

Real per-capita consumption growth rate:

$$\Delta C_t = EX_{\Delta N,t} + g_z - 1 + \hat{g}_{z,t} + \hat{c}_t - \hat{c}_{t-1} + EX_{\Delta C,t}. \tag{292}$$

Real per-capita investment growth rate (excluding inventories):

$$\Delta I_{NI,t} = EX_{\Delta N,t} + g_z - 1 + \hat{g}_{z,t} + \hat{\imath}_t - \hat{\imath}_{t-1} + EX_{\Delta INI,t}. \tag{293}$$

Real per-capita government consumption growth rate:

$$\Delta G_t = EX_{\Delta N,t} + g_z - 1 + \hat{g}_{z,t} + \hat{g}_t - \hat{g}_{t-1} + EX_{\Delta G,t}. \tag{294}$$

Real per-capita export growth:

$$\Delta X_t = EX_{\Delta N,t} + g_z - 1 + \hat{g}_{z,t} + \hat{x}_t - \hat{x}_{t-1} + EX_{\Delta X,t}. \tag{295}$$

Real per-capita import growth rate:

$$\Delta IM_t = EX_{\Delta N,t} + g_z - 1 + \hat{g}_{z,t} + \widehat{im}_t - \widehat{im}_{t-1} + EX_{\Delta IM,t}. \tag{296}$$

Inflation in market price GDP deflator:

$$\Delta P^M_{Y,\,t} = \hat{\epsilon}_{OB\_DPY,t} + \hat{\pi}_{C,t} + \hat{p}^M_{Y,t} - \hat{p}^M_{Y,t-1} + \pi - 1 - \hat{\bar{\pi}}_t. \tag{297}$$

Inflation in factor price CPI (Excluding VAT, fruits and vegetables):

$$\Delta P_{C,t} = \hat{\pi}_{C,t} + \pi - 1 - \bar{\pi}_t. \tag{298}$$

Inflation in investment deflator (including a measurement error):

$$\Delta P_{I,t} = \pi - 1 + \hat{\pi}_{C,t} + \hat{p}_{I,t} - \hat{p}_{I,t-1} - \bar{\pi}_t + \eta_t^{\Delta P_I}. \tag{299}$$

Annualized inflation target:

$$4\overline{\pi}^*_t = 4\,(\bar{\pi}_t + \pi - 1)\,. \tag{300}$$

Per capita employment (deviation from HP trend):

$$\hat{e}_t = \frac{\beta}{1+\beta\,\chi_E}\,E_t(\hat{e}_{t+1}) + \frac{\chi_E}{1+\beta\,\chi_E}\,\hat{e}_{t-1} + \frac{(1-\beta\,\xi_E)\,(1-\xi_E)}{(1+\beta\,\chi_E)\,\xi_E}\,(\hat{n}_t - \hat{e}_t) + \hat{\epsilon}_{OB\_E,t}. \tag{301}$$

Per capita employment:

$$\Delta EM_t = EX_{\Delta N,t} + \hat{e}_t - \hat{e}_{t-1}. \tag{302}$$

Per capita labor input:

$$\Delta N_t = EX_{\Delta N,t} + \hat{n}_t - \hat{n}_{t-1}. \tag{303}$$

Nominal wage growth rate :

$$\Delta W_t = \pi - 1 + g_z - 1 + \hat{\pi}_{C,t} + \hat{g}_{z,t} + \hat{w}_t - \hat{w}_{t-1} - \bar{\pi}_t + EX_{\Delta W,t}. \tag{304}$$

Annualized nominal interest rate :

$$r_t^{OB} = 4\,(\hat{r}_t + r - 1) -\, 4\bar{\pi}_t. \tag{305}$$

Nominal depreciation rate:

$$\Delta S_t = \hat{c}_t - \hat{c}_{t-1} + \hat{\pi}_{Y,t} - \hat{\pi}^*_{Y,t} + \pi - 1 - (\pi^* - 1) - \bar{\pi}_t + EX_{\Delta S,t}. \tag{306}$$

Foreign output growth rate:

$$\Delta Y^*{}_t = g_z - 1 + \hat{g}_{z,t} + \hat{\tilde{z}}_t + \hat{y}^*_t - \hat{y}^*_{t-1} - \hat{\tilde{z}}_{t-1} + EX_{\Delta Y*,t}. \tag{307}$$

Inflation in foreign price deflator:

$$\Delta P_{Y^*,t} = \hat{\pi}^*_{Y,t} + \pi^* - 1. \tag{308}$$

Annualized foreign nominal interest rate:

$$R^{*OB}_t = 4\left(\hat{r}^*_t + r^* - 1\right). \tag{309}$$

Inflation in foreign competitors price:

$$\Delta P_{X^*,t} = \Delta P_{Y^*,t} + \hat{p}^*_{x,t} - \hat{p}^*_{x,t-1}. \tag{310}$$

Inflation in export deflator (NIS terms):

$$\Delta P^{NIS}_{X,t} = \Delta S_t + \Delta P_{Y^*,t} + \hat{p}_{x,t} - \hat{p}_{x,t-1}. \tag{311}$$

Forward long run expected real rate (5-10 years):

$$rr^{fwd\_ob}_t = \hat{\epsilon}_{fwd\_ob,t} + 4\left(\hat{fw}_{ri^*,t} + \frac{g_z}{\beta} - 1\right) + tp, \tag{312}$$

where tp captures an average term premium.

Observable ratio of Current Account to GDP:

$$S_{CA,t} = (CA/Y)_t. \tag{313}$$

Observable consumption tax rate:

$$\tau^{C\_OB}_t = \tau_c + \hat{\tau}^c_t. \tag{314}$$

Observable income tax rate :

$$\tau_t^{N\_OB} = \tau_n + \hat{\tau}_t^n. \tag{315}$$

Observable change in oil prices:

$$\Delta P_{OIL,t} = \hat{p}_{OIL,t} - \hat{p}_{OIL,t-1} + \Delta P_{Y^*,t} + EX_{\Delta POIL,t}. \tag{316}$$

The change in inventories, as a share of GDP :

$$\Delta inv_t = \Delta\widehat{inv}_t + \Delta inv + EX_{\Delta INV,t}. \tag{317}$$

Forward long run expected nominal rate abroad (5-10 years):

$$r_t^{*fwd\_ob} = \hat{\epsilon}_{fwd\_ob^*,t} + 4\left(\bar{\pi}_t^* + \widehat{fw}_{ri^*,t} + r^* - 1\right) + tp^*, \tag{318}$$

where $tp*$ captures an average term premium.

Observable world import growth rate:

$$\Delta WT_t^* = \hat{g}_{z,t} + \hat{\tilde{z}}_t + \widehat{im}_t^* - \widehat{im}_{t-1}^* - \hat{\tilde{z}}_{t-1} + g_z - 1 + EX_{\Delta WT*,t}. \tag{319}$$

D.11 AR(1) processes of the idiosyncratic trend shocks

$$EX_{\Delta S,t} = \rho_{EX}^{\Delta S}\, EX_{\Delta S,t-1} + \eta_{EX,t}^{S}. \tag{320}$$

$$EX_{\Delta W,t} = \left(1 - \rho_{EX}^{\Delta W}\right)\left(g_{\Delta W} - g_z - (\pi - 1)\right) + \rho_{EX}^{\Delta W}\, EX_{\Delta W,t-1} + \eta_{EX,t}^{W}. \tag{321}$$

$$EX_{\Delta N,t} = \left(1 - \rho_{EX}^{N}\right) g_{\Delta N} + \rho_{EX}^{N}\, EX_{\Delta N,t-1} + \eta_{EX,t}^{N}. \tag{322}$$

$$EX_{\Delta C,t} = \left(1 - \rho_{EX}^{C}\right)\left(g_{\Delta C} - g_z - g_{\Delta N}\right) + \rho_{EX}^{C}\, EX_{\Delta C,t-1} + \eta_{EX,t}^{C}. \tag{323}$$

$$EX_{\Delta INI,t} = \left(1-\rho^{I}_{EX}\right)\,\left(g_{\Delta I} - g_z - g_{\Delta N}\right) + \rho^{I}_{EX}\,EX_{\Delta INI,t-1} + \eta^{I}_{EX,t}. \tag{324}$$

$$EX_{\Delta IM,t} = \left(1-\rho^{IM}_{EX}\right)\,\left(g_{\Delta IM} - g_z - g_{\Delta N}\right) + \rho^{IM}_{EX}\,EX_{\Delta IM,t-1} + \eta^{IM}_{EX,t}. \tag{325}$$

$$EX_{\Delta X,t} = \left(1-\rho^{X}_{EX}\right)\,\left(g_{\Delta X} - g_z - g_{\Delta N}\right) + \rho^{X}_{EX}\,EX_{\Delta X,t-1} + \eta^{X}_{EX,t}. \tag{326}$$

$$EX_{\Delta INV,t} = \eta^{\Delta INV}_{EX,t}. \tag{327}$$

$$0 = \frac{c}{y}EX_{\Delta C,t} + \frac{i}{y}EX_{\Delta INI,t} + \Delta inv\,EX_{\Delta INV,t} + s_G\,EX_{\Delta G,t} + s_X EX_{\Delta X,t} - \frac{im}{y}EX_{\Delta IM,t}. \tag{328}$$

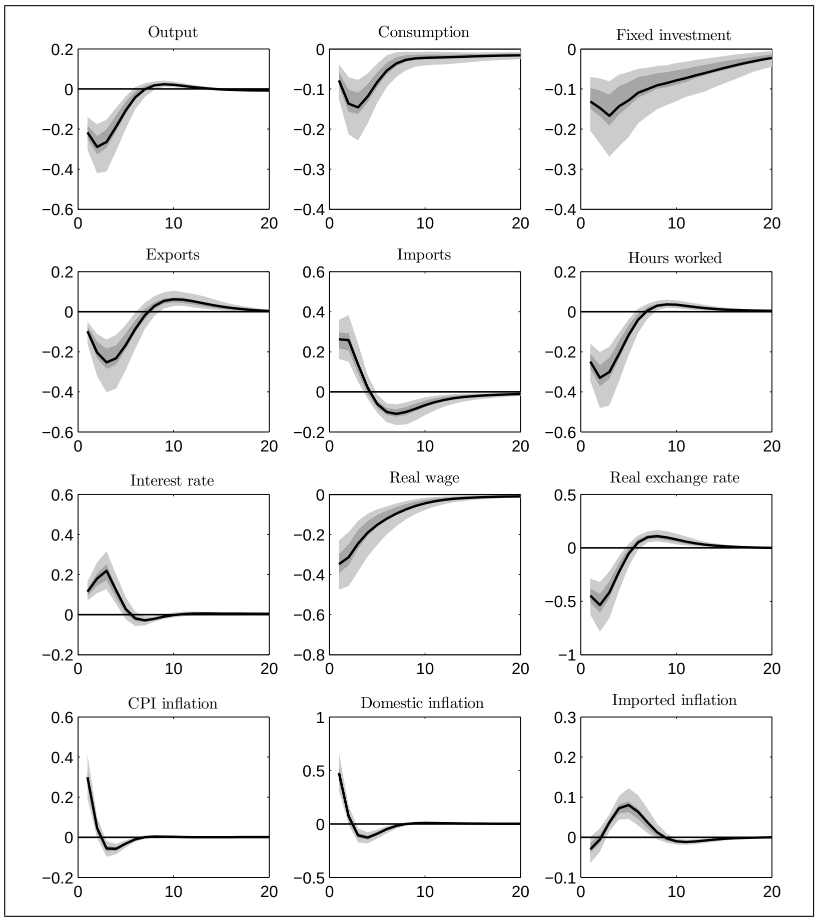
$$EX_{\Delta Y*,t} = \left(1-\rho^{Y^*}_{EX}\right)\,\left(g_{\Delta Y^*} - g_z\right) + \rho^{Y^*}_{EX}\,EX_{\Delta Y*,t-1} + \eta^{Y^*}_{EX,t}. \tag{329}$$

$$EX_{\Delta WT*,t} = \left(1-\rho^{WT^*}_{EX}\right)\,\left(g_{\Delta WT*} - g_z\right) + \rho^{WT^*}_{EX}\,EX_{\Delta WT*,t-1} + \eta^{WT^*}_{EX,t}. \tag{330}$$

$$EX_{\Delta POil,t} = \rho^{P^*_{OIL}}_{EX}\,EX_{\Delta POil,t-1} + \eta^{P^*_{OIL}}_{EX,t}. \tag{331}$$

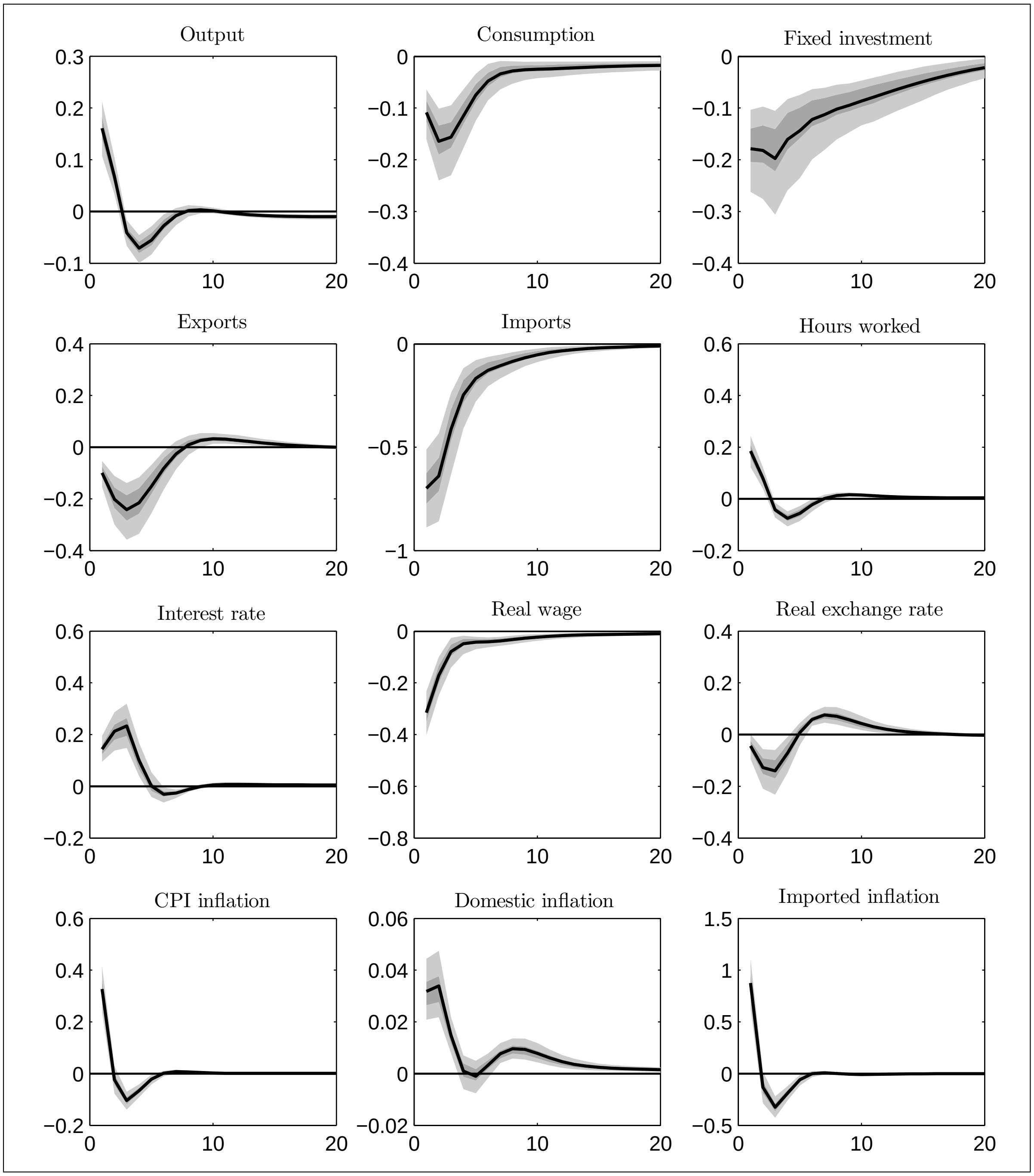
Appendix E Bayesian impulse responses

Figure 11: Impulse Response to a Domestic Price Markup Shock



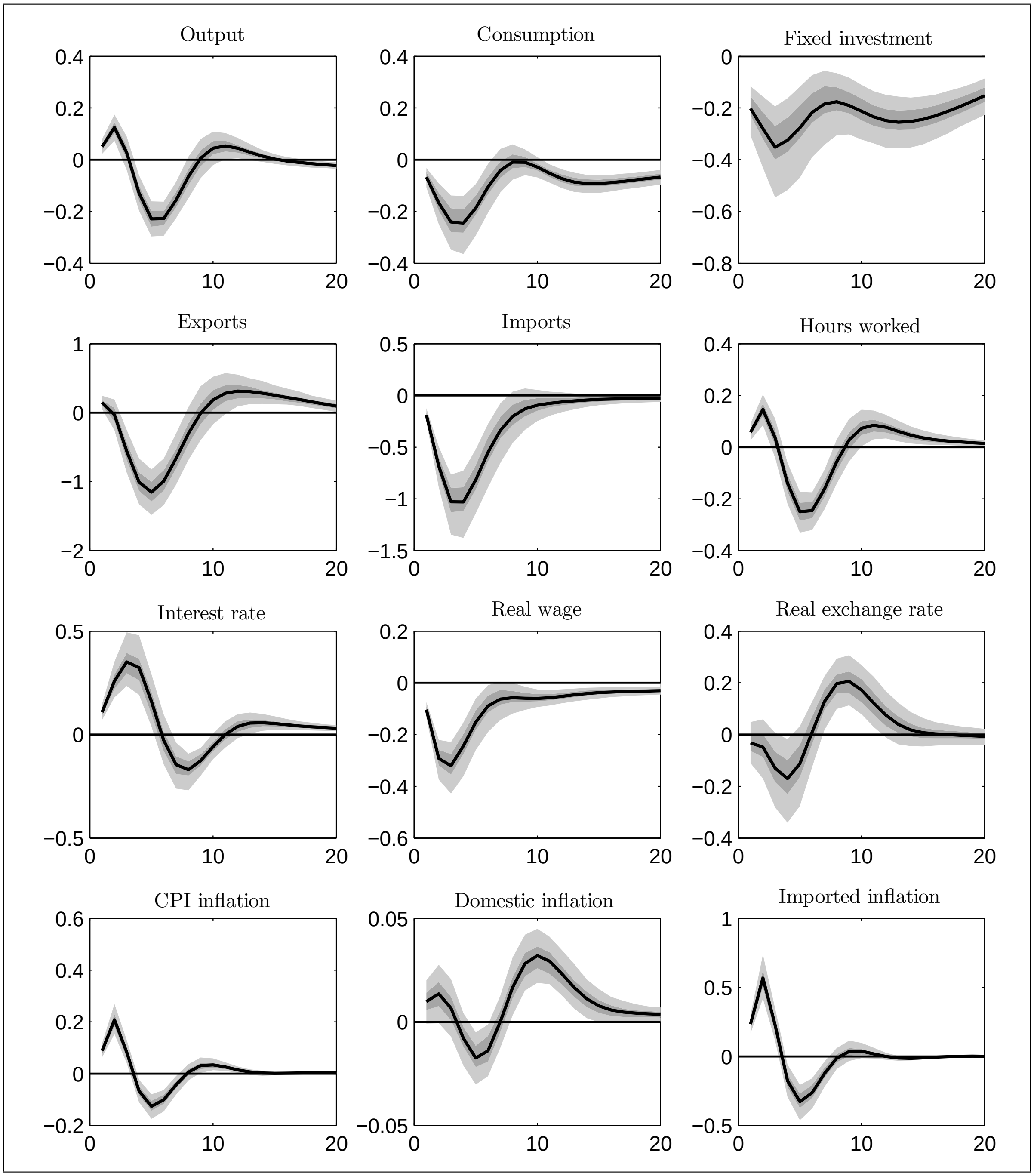
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 12: Impulse Response to an Import Price Markup Shock



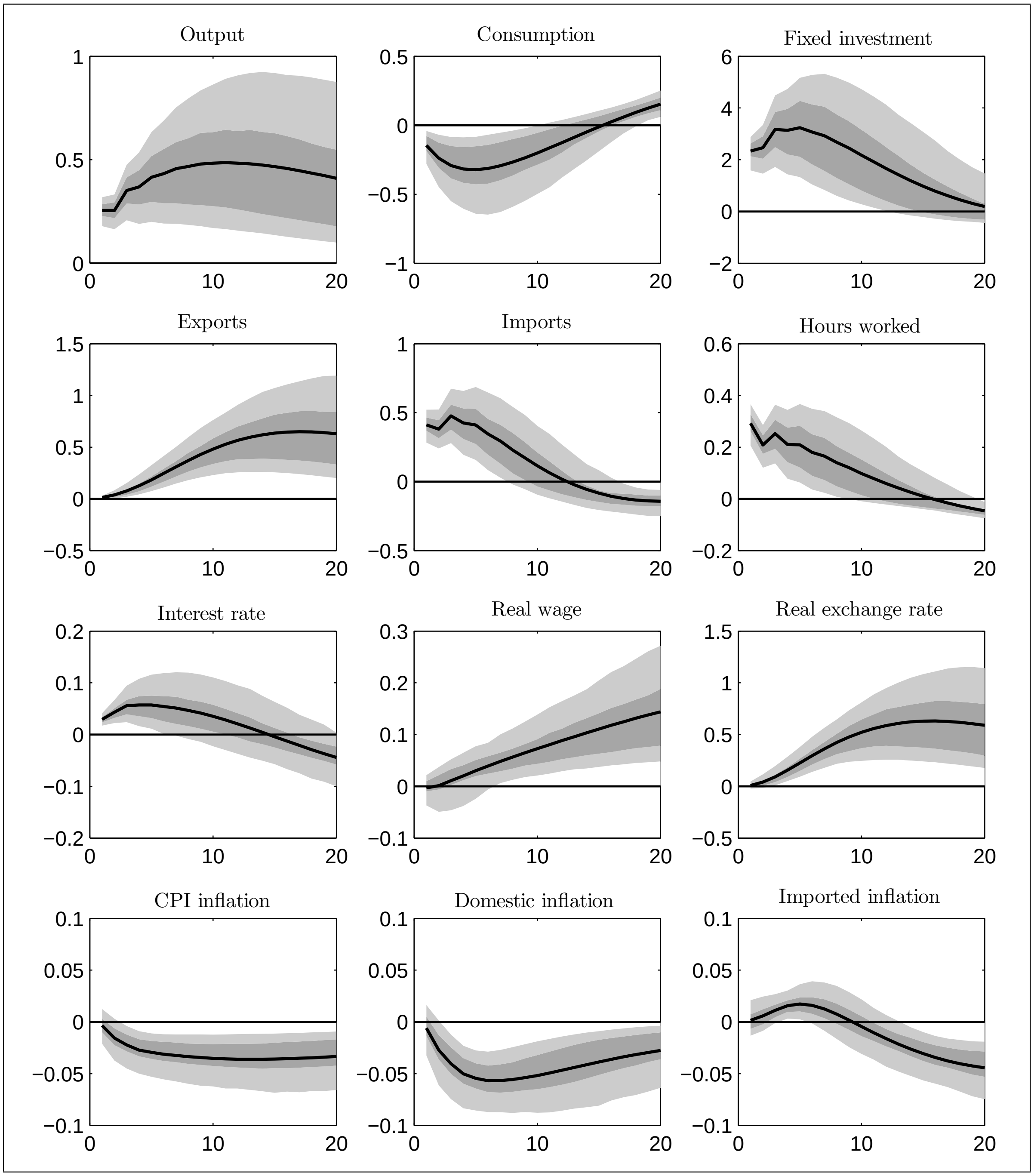
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 13: Impulse Response to an Oil Price Shock



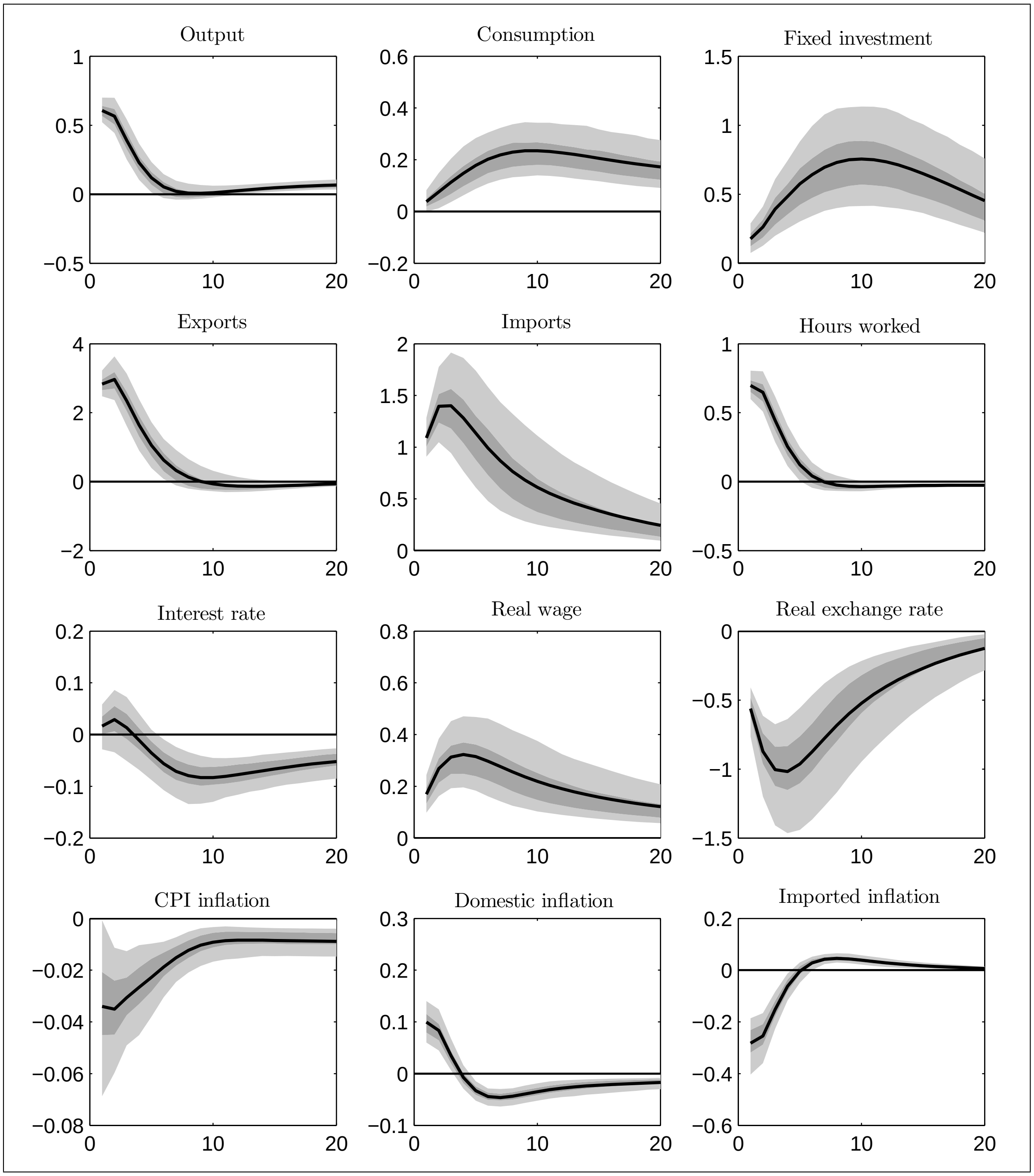
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 14: Impulse Response to an Investment Specific Technology Shock



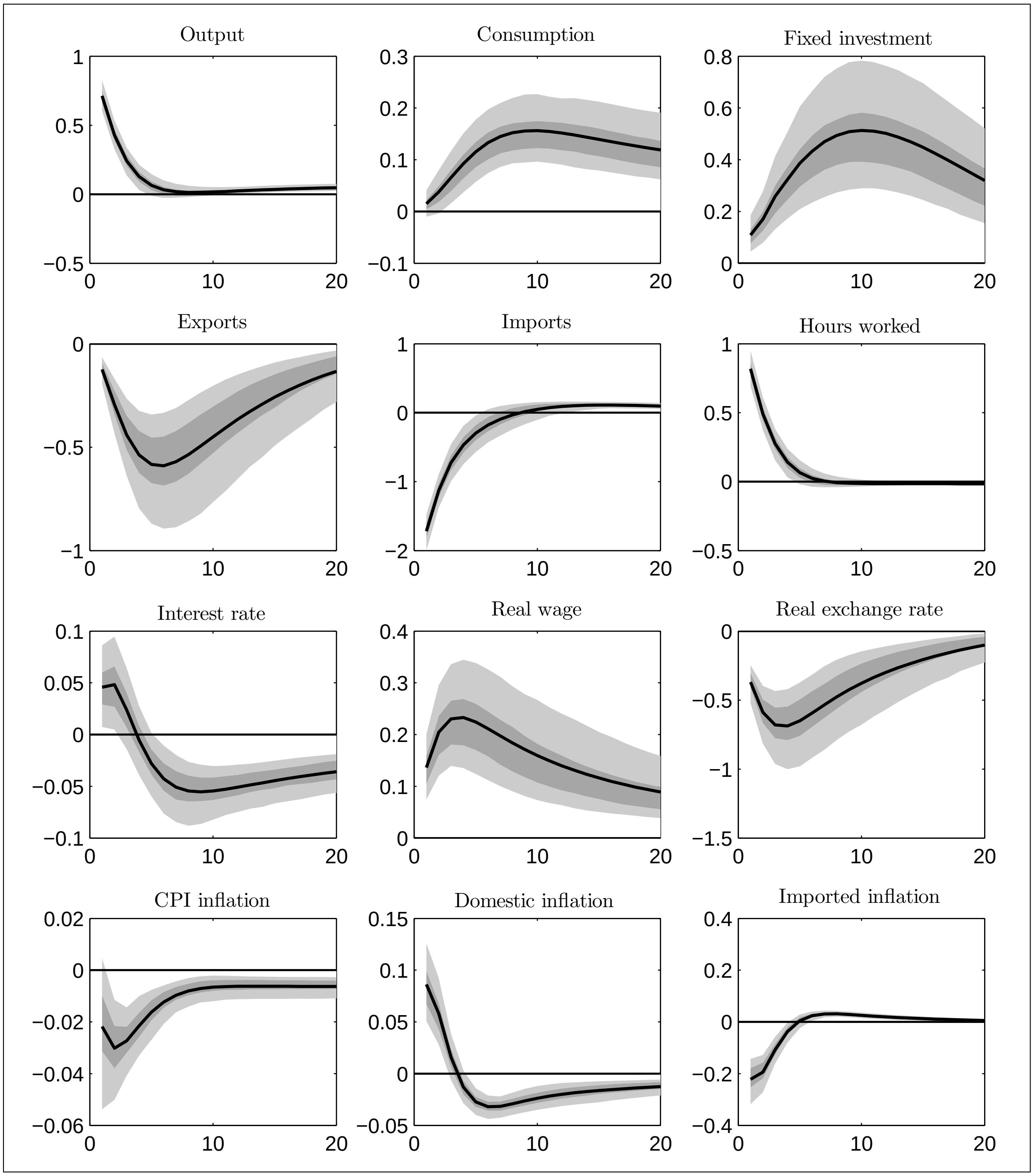
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 15: Impulse Response to an Export Share Shock



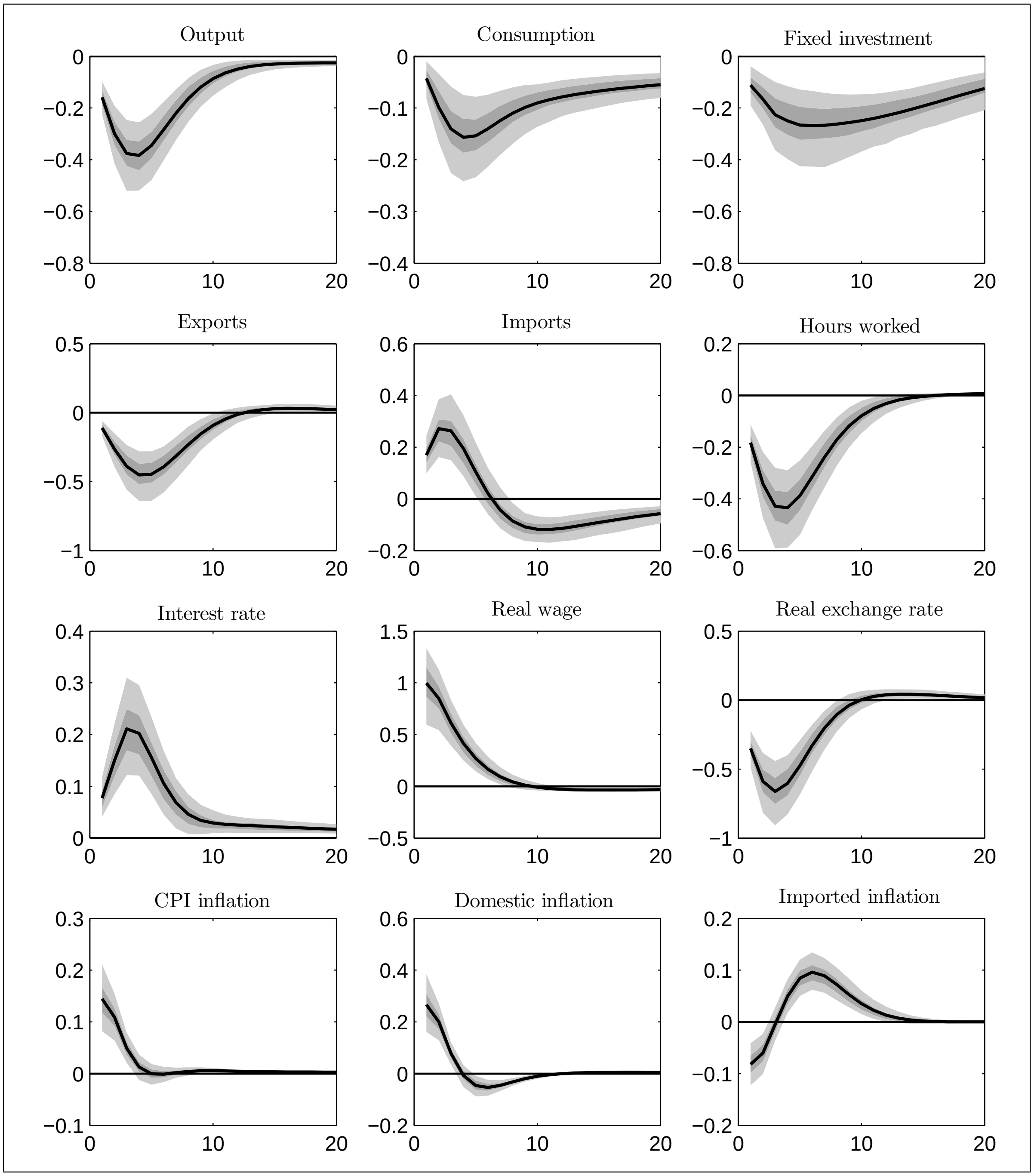
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 16: Impulse Response to a Home Bias Shock



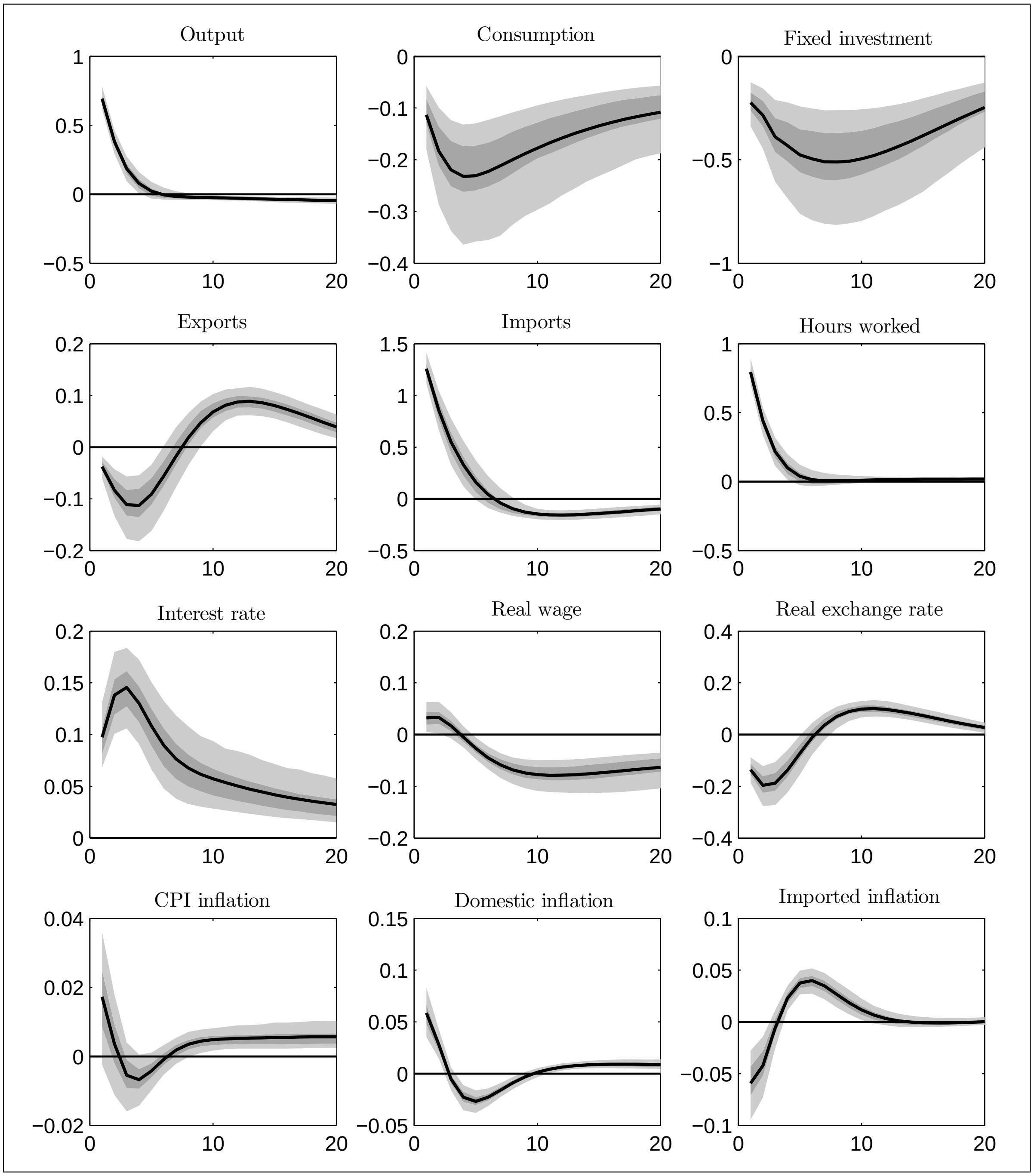
Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 17: Impulse Response to a Wage Markup Shock



Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

Figure 18: Impulse Response to an Inventories Investment Demand Shock



Note: Shock of one standard deviation. Solid line – mean of impulse response. Gray area – 70 and 90 percent highest interval of impulse response. Real variables – percentage deviation from steady state. Inflation – percentage point deviation from steady state. Interest rate – annualized percentage point deviation from steady state.

References

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011). Dynare: Reference Manual, Version 4. *Dynare Working Papers*.

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007). Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through. *Journal of International Economics 72*(2), 481–511.

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2008). Evaluating an Estimated New Keynesian Small Open Economy Model. *Journal of Economic Dynamics and Control 32*(8), 2690–2721.

An, S. and F. Schorfheide (2007). Bayesian Analysis of DSGE Models. *Econometric Reviews 26*(2-4), 113–172.

Argov, E., A. Binyamini, D. Elkayam, and I. Rozenshtrom (2007). A Small Macroeconomic Model to Support Inflation Targeting in Israel. *Bank of Israel, Monetary Department*.

Argov, E. and D. Elkayam (2010). An Estimated New Keynesian Model for Israel. *Israel Economic Review 7*(2), 1–40.

Beenstock, M. and A. Ilek (2010). Wicksell's Classical Dichotomy: Is the Natural Rate of Interest Independent of the Money Rate of Interest? *Journal of Macroeconomics 32*(1), 366–377.

Beneš, J., A. Binning, M. Fukac, K. Lees, and T. Matheson (2009). *KITT: Kiwi Inflation Targeting Technology.* Reserve Bank of New Zealand.

Binyamini, A. (2007). Small Open Economy New Keynesian Phillips Curve: Derivation and Application to Israel. *Israel Economic Review 5*(1), 67–92.

Binyamini, A., Z. Eckstein, and K. Flug (2008). The Evolution of Monetary-Policy Strategy and Exchange-Rate Regime in Israel. In *ECB workshop on Economic and Financial Developments in Mediterranean Countries, 2008*.

Brooks, S. and A. Gelman (1998). General Methods for Monitoring Convergence of Iterative Simulations. *Journal of Computational and Graphical Statistics 7*(4), 434–455.

Brubakk, L., T. Husebo, J. Maih, K. Olsen, and M. Ostnor (2006). Finding NEMO: Documentation of the Norwegian Economy Model. Technical Report 2006/6, Norges Bank Staff Memo.

Calvo, G. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of monetary Economics 12*(3), 383–398.

Canova, F. (2009). Bridging Cyclical DSGE Models and the Raw Data. Manuscript, Centre de Recerca en Economia Internacional.

Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy 113*(1), 1–45.

Christiano, L., M. Trabandt, and K. Walentin (2007). Introducing Financial Frictions and Unemployment into a Small Open Economy Model. *Manuscript, Sveriges Riksbank Working Paper Series*.

Christoffel, K., G. Coenen, and A. Warne (2008). The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis. *European Central Bank, Working Paper Series 944*.

Dixit, A. and J. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity. *The American Economic Review 67*(3), 297–308.

Eckstein, Z. and T. Ramot-Nyska (2008). Twenty Years of Financial Liberalisation in Israel: 1987–2007. In *BIS Papers No 44, Financial Globalisation and Emerging Market Capital Flows*, pp. 289.

Elkayam, D. (2003). The Long Road from Adjustable Peg to Flexible Exchange Rate Regimes: The Case of Israel. *Bank of Israel discussion Paper 2003.04*.

Friedman, A. and Y. Lavi (2007). Israel's Real Exchange Rate and Foreign Trade. *Bank of Israel Review 79*(Hebrew), 37–86.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-Run Implications of Investment-Specific Technological Change. *The American Economic Review 87*(3), 342–362.

Juillard, M. (1996). Dynare: A Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm. *CEPREMAP Working Papers* (9602).

Kydland, F. and E. Prescott (1982). Time to Build and Aggregate Fluctuations. *Econometrica 50*(6), 69–85.

Leiderman, L. and H. Bar-Or (2002). Monetary Policy Rules and Transmission Mechanisms Under Inflation Targeting in Israel. In Loayza and Smith-Hebbel (Eds.), *Monetary policy: rules and transmission mechanism*. Banco Central de Chile, Santiago.

Lucas, R. (1976). Econometric Policy Evaluation: A Critique. In *Carnegie-Rochester conference series on public policy*, Volume 1, pp. 19–46. Elsevier.

Maih, J. (2010). Conditional Forecasts in DSGE Models. *Norges Bank Working Paper* (2010/07).

Melnick, R. (2005). A Peek into the Governor's Chamber: The Israeli Case. *Israel Economic Review 3*(1), 1–21.

Murchison, S. and A. Rennison (2006). ToTEM: The Bank of Canada's New Quarterly Projection Model. *Bank of Canada Technical Report 97.*

Ribon, S. (2004). A New Phillips Curve for Israel. *Bank-of-Israel Discussion Paper* (2004.11).

Schmitt-Grohe, S. and M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics 61*(1), 163–185.

Schorfheide, F. (2011). Estimation and Evaluation of DSGE Models: Progress and Challenges. *Federal Reserve Bank of Philadelphia, Research Department Working Paper 11-7.*

Seneca, M. (2010). A DSGE Model for Iceland. *Central Bank of Iceland, Working Paper 50.*

Smets, F. and R. Wouters (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association 1*(5), 1123–1175.

Sussman, N. (2007). Monetary Policy in Israel During 1990-2000: Estimating the Central Bank Response Function. In N. Liviatan and H. Barkai (Eds.), *The Bank of Israel – Selected Topics in Israel's Monetary Policy*, Volume 2, pp. 55–78. Oxford University Press.

Taylor, J. (1993). Discretion versus Policy Rules in Practice. In *Carnegie-Rochester conference series on public policy*, Volume 39, pp. 195–214. Elsevier.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton Univ Press.